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Slide of the Seminar

Phase diagram of turbulent Taylor-Couette flow

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Phase diagram of turbulent Taylor-Couette flow

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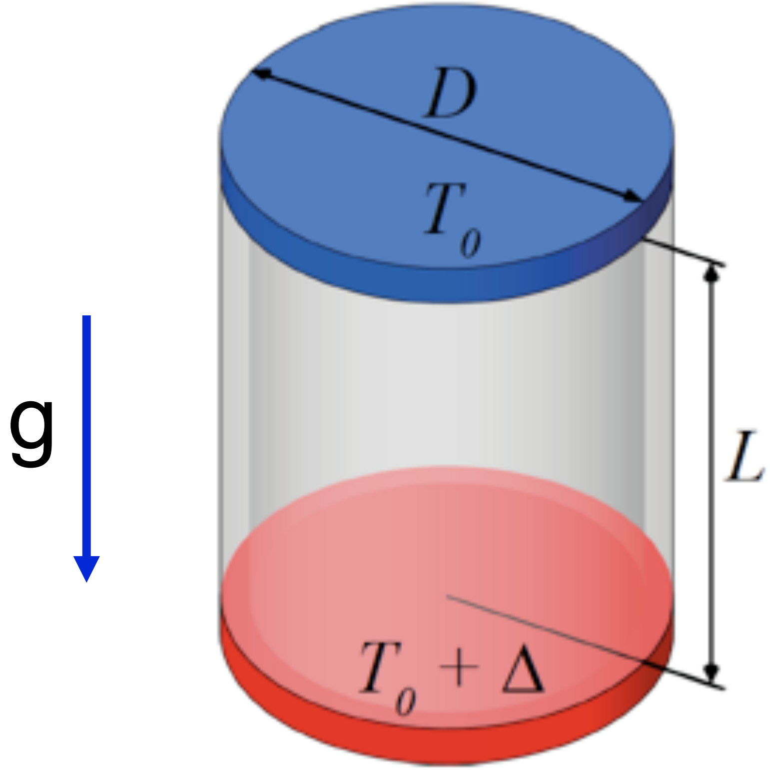
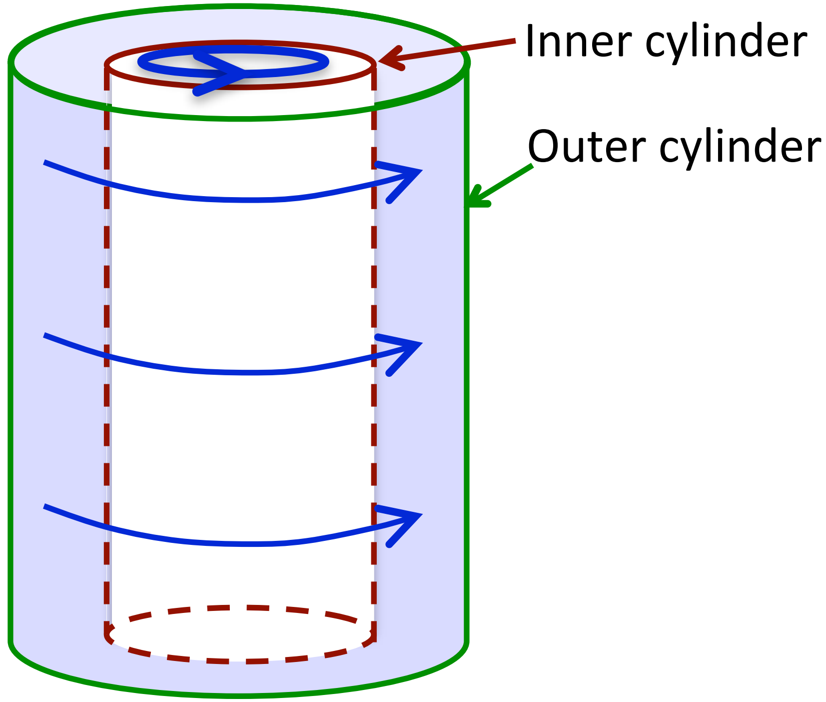
Siegfried Grossmann
Detlef Lohse

Physics of Fluids
University of Twente.

“Drosophila” of Physics of Fluids





Rayleigh-Bénard:

Heat transfer

Taylor-Couette:

Angular velocity transfer

-- closed systems -- global balances -- mathematically well defined

Twente Turbulent Taylor-Couette (T^3C)

van Gils, Bruggert, Lathrop, Sun & Lohse, Rev. Sci. Instrum. 82, 025105 (2011)

Twente Turbulent Taylor-Couette (T^{3}C)

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Rev. Sci. Instrum. 82, 025105 (2011)

Twente Turbulent Taylor-Couette (T^3C)

Ideal systems to study turbulent boundary layer - bulk interaction

Physics

Physics **5**, 4 (2012)

Viewpoint

The Twins of Turbulence Research

Friedrich H. Busse
Physikalisches Institut der Universität Bayreuth, D-95440 Bayreuth, Germany

Published January 9, 2012

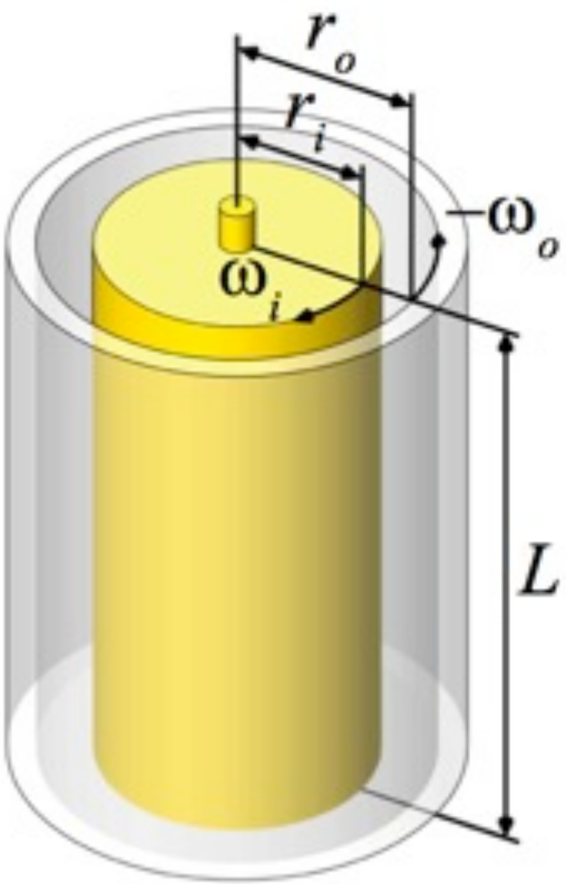
Two new experiments on fluid turbulence have attained conditions needed to establish asymptotic scalings for turbulent transports of heat and angular momentum.

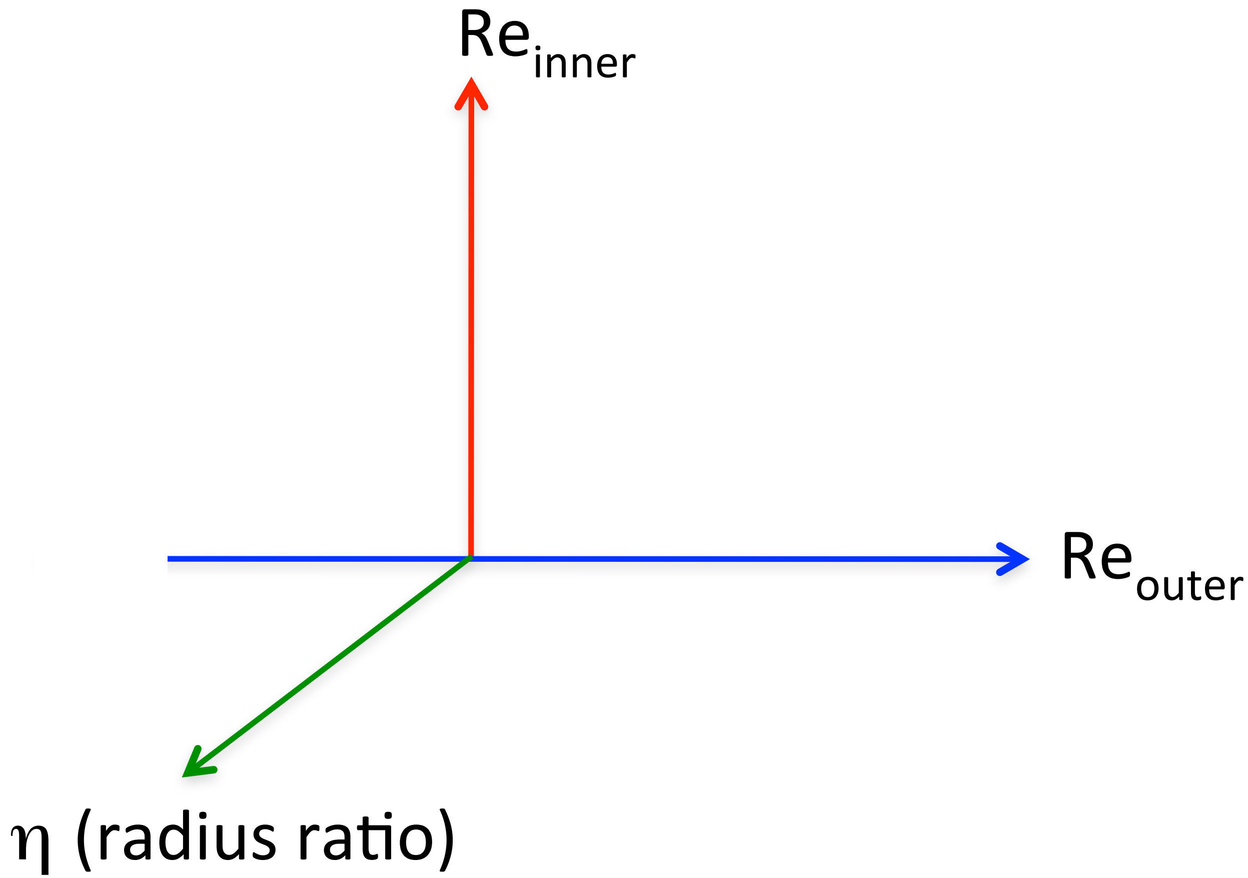
Subject Areas: **Fluid Dynamics**

A Viewpoint on:
Transition to the Ultimate State of Turbulent Rayleigh-Bénard Convection
Xiaozhou He, Denis Funfschilling, Holger Nobach, Eberhard Bodenschatz, and Guenter Ahlers
Phys. Rev. Lett. **108**, 024502 (2012) – Published January 9, 2012

Ultimate Turbulent Taylor-Couette Flow
Sander G. Huisman, Dennis P. M. van Gils, Siegfried Grossmann, Chao Sun, and Detlef Lohse
Phys. Rev. Lett. **108**, 024501 (2012) – Published January 9, 2012

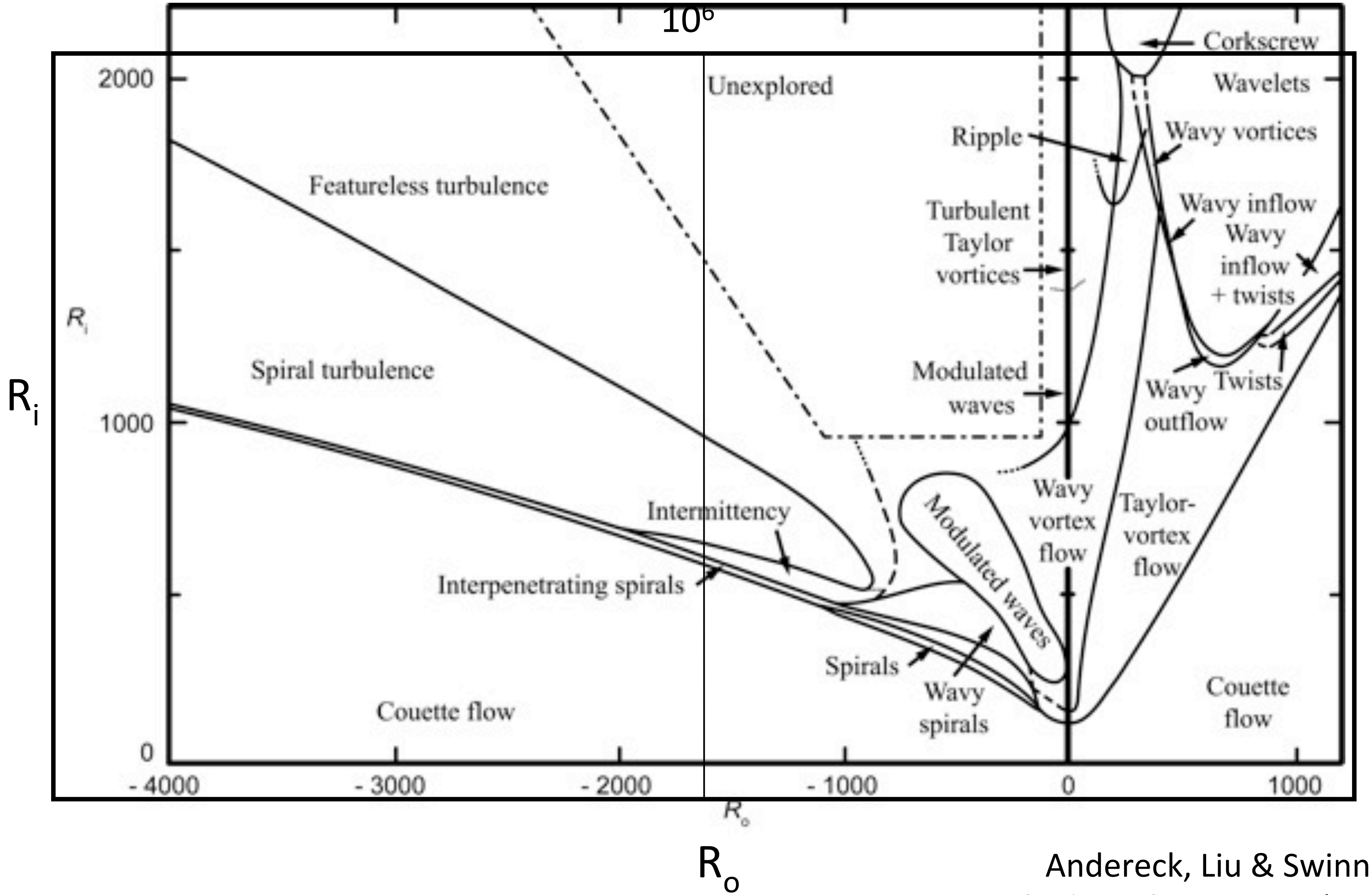
Taylor-Couette: parameter space





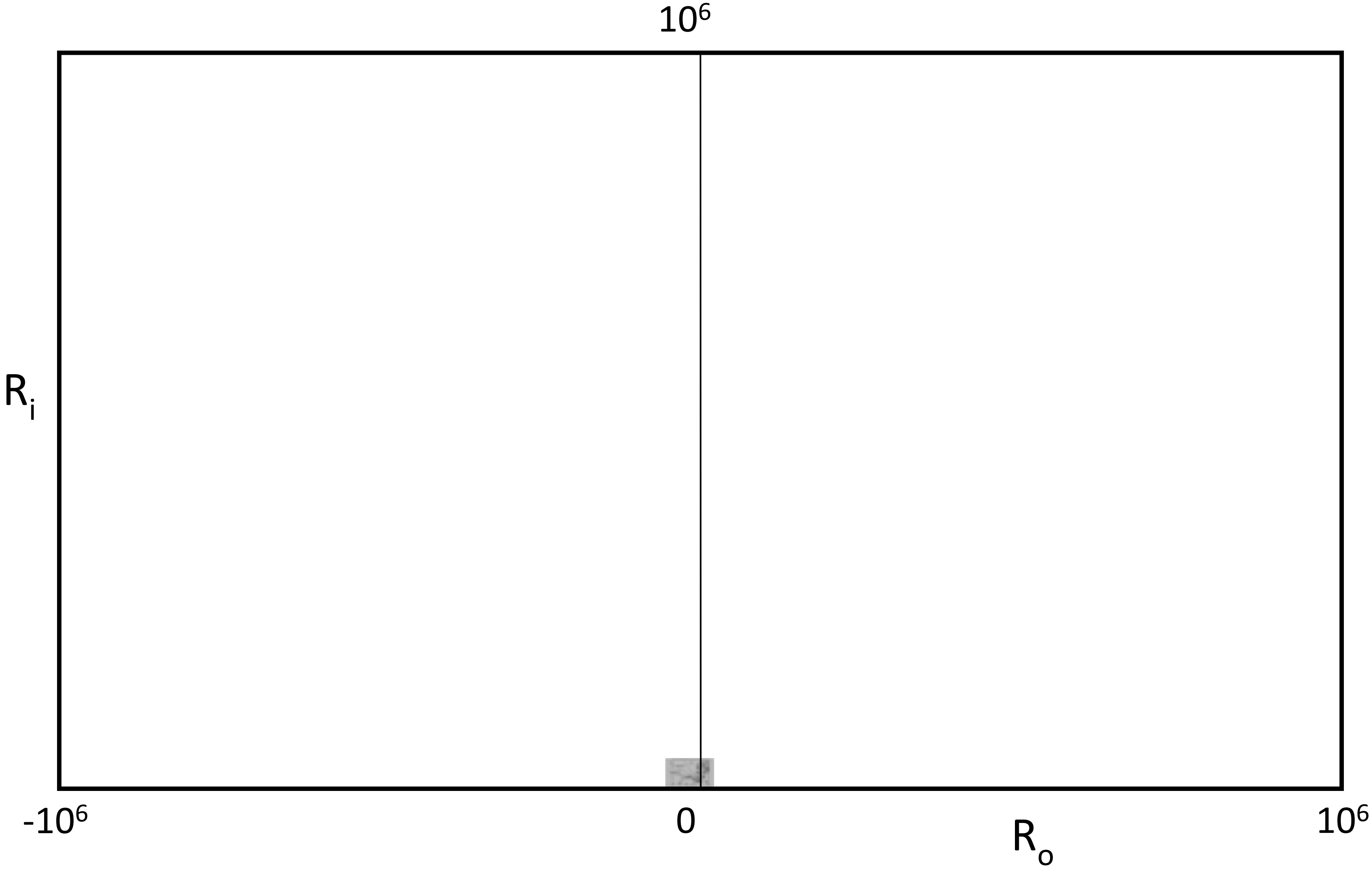
Parameter space of T^3C

Rich flow structures

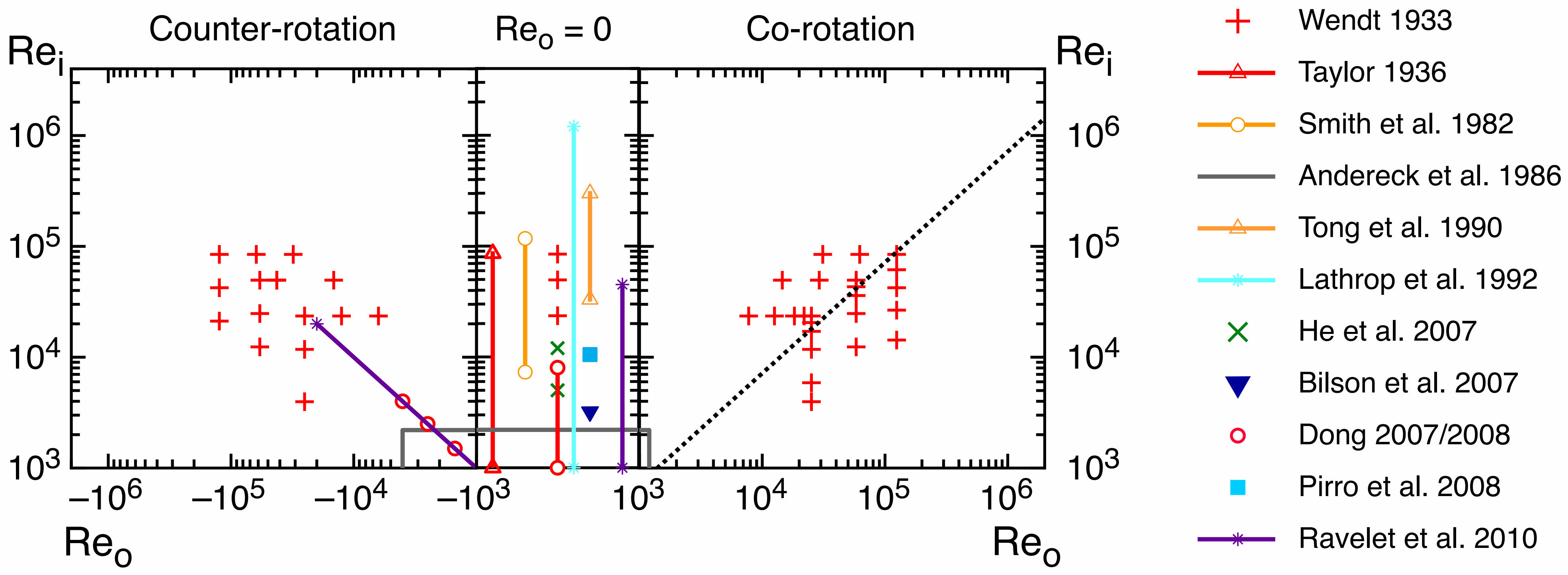


Andereck, Liu & Swinney,
J. Fluid Mech. 164, 155 (1986)

Parameter space of T^3C



Experimentally explored parameters ~2010



T^3C: parameter space

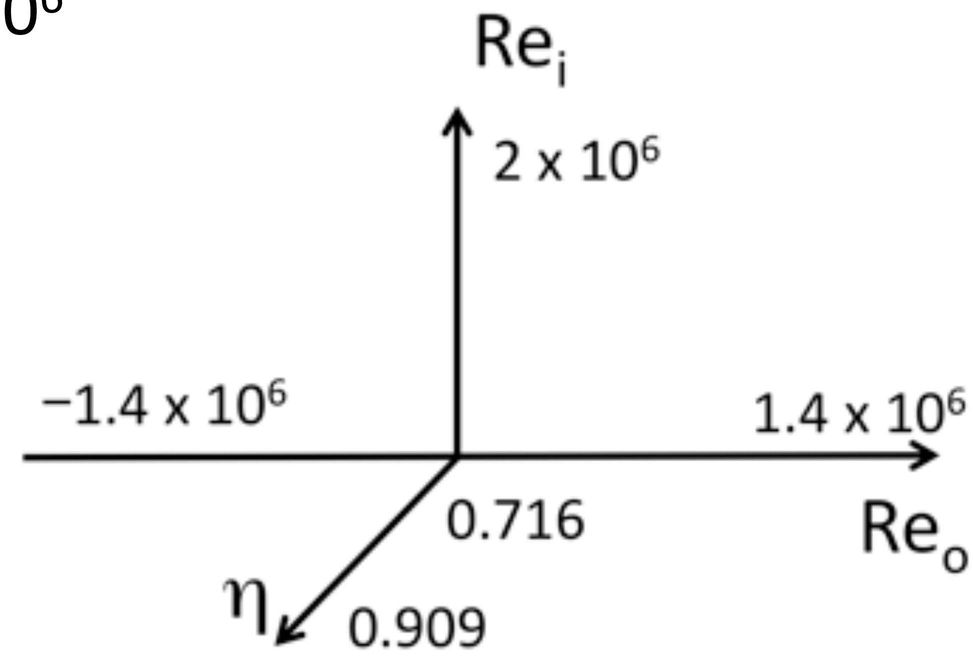
- Independently rotating cylinders

  IC 20 Hz, OC 10 Hz
- Max. Reynolds number

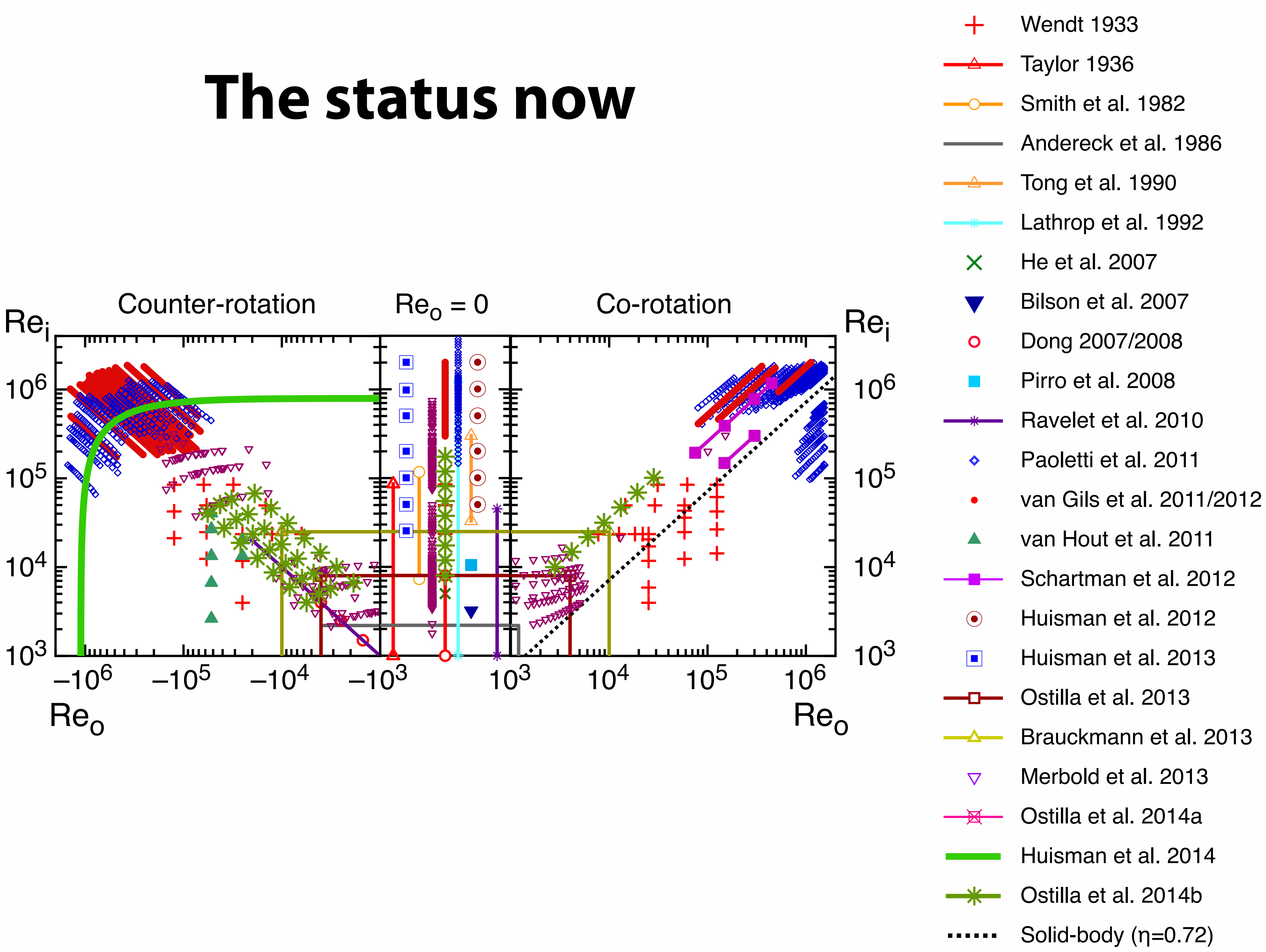
  counter rotation: 3.4×10^6

  pure IC rotation: 2.0×10^6

  pure OC rotation: 1.4×10^6
- Variable radius ratio η
- 111 liters of liquid
- 1K/minute heating!!
- 20 kW cooling

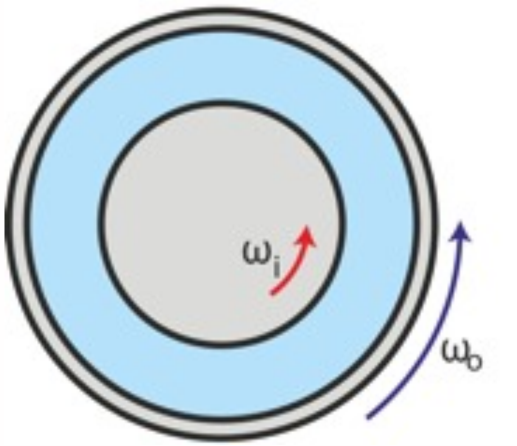


The status now



Navier-Stokes equation & BCs

Lab frame



$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nu \nabla^2 \mathbf{u}$$

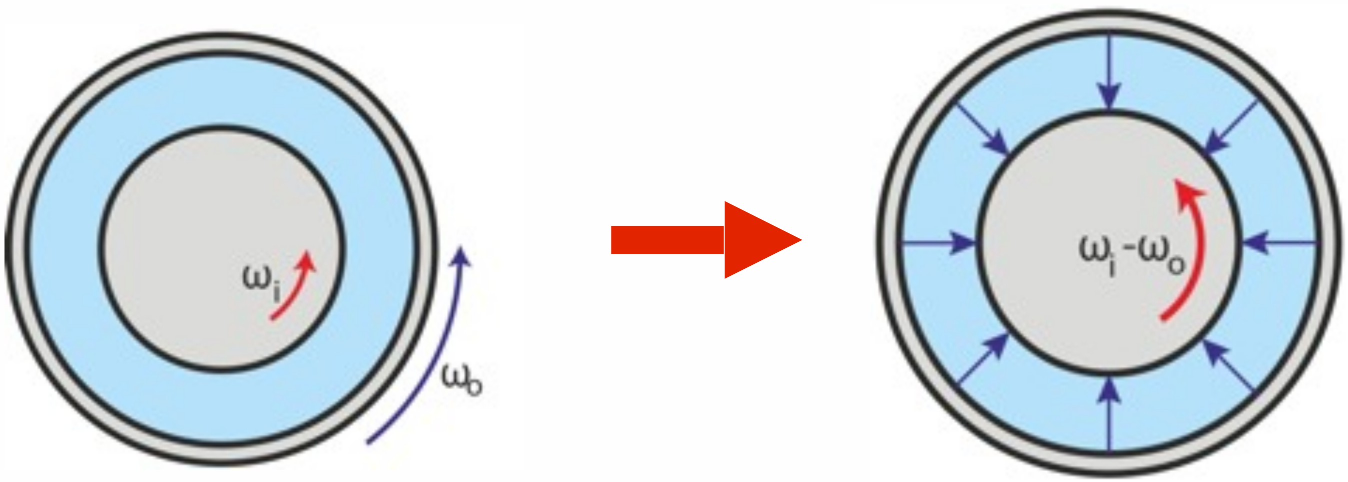
$$U_\theta(r_i) = r_i \omega_i$$

$$U_\theta(r_o) = r_o \omega_o$$

Change of frame of reference: Outer cylinder rotation as Coriolis force

Lab frame

Rotating frame



$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nu \nabla^2 \mathbf{u}$$

$$U_\theta(r_i) = r_i \omega_i$$

$$U_\theta(r_o) = r_o \omega_o$$

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} + 2\omega_o \times \mathbf{u} = -\nabla p + \nu \nabla^2 \mathbf{u}$$

$$U_\theta(r_i) = r_i(\omega_i - \omega_o) \equiv U$$

$$U_\theta(r_o) = 0$$

$$d = r_o - r_i$$

Outer cylinder rotation as Coriolis force

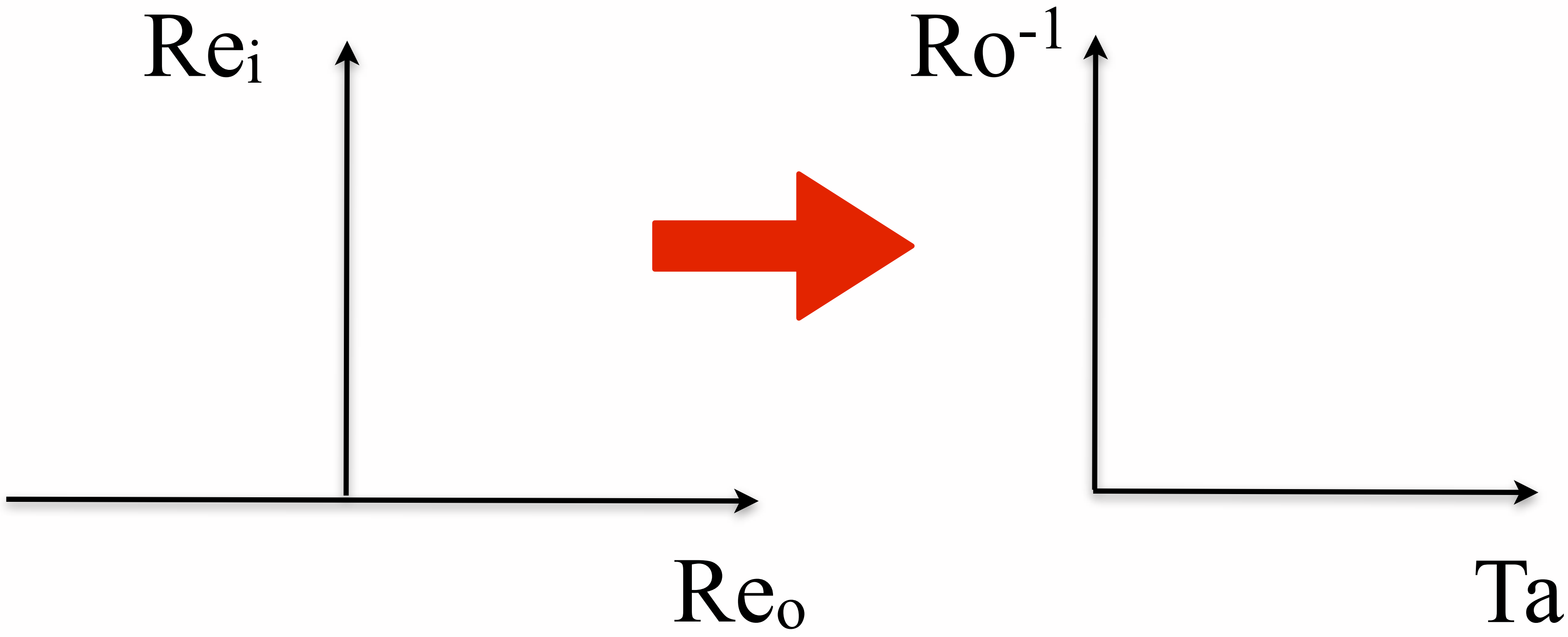
Navier-Stokes equation in dimensionless form:

$$\partial_t \hat{\mathbf{u}} + \hat{\mathbf{u}} \cdot \hat{\nabla} \hat{\mathbf{u}} + Ro^{-1} \mathbf{e}_z \times \hat{\mathbf{u}} = -\hat{\nabla} \hat{p} + Re_s^{-1} \hat{\nabla}^2 \hat{\mathbf{u}}$$

$$Ro^{-1} = \frac{2\omega_o (r_o - r_i)}{r_i(\omega_i - \omega_o)} \qquad Re_s = \frac{r_i(\omega_i - \omega_o)(r_o - r_i)}{\nu}$$

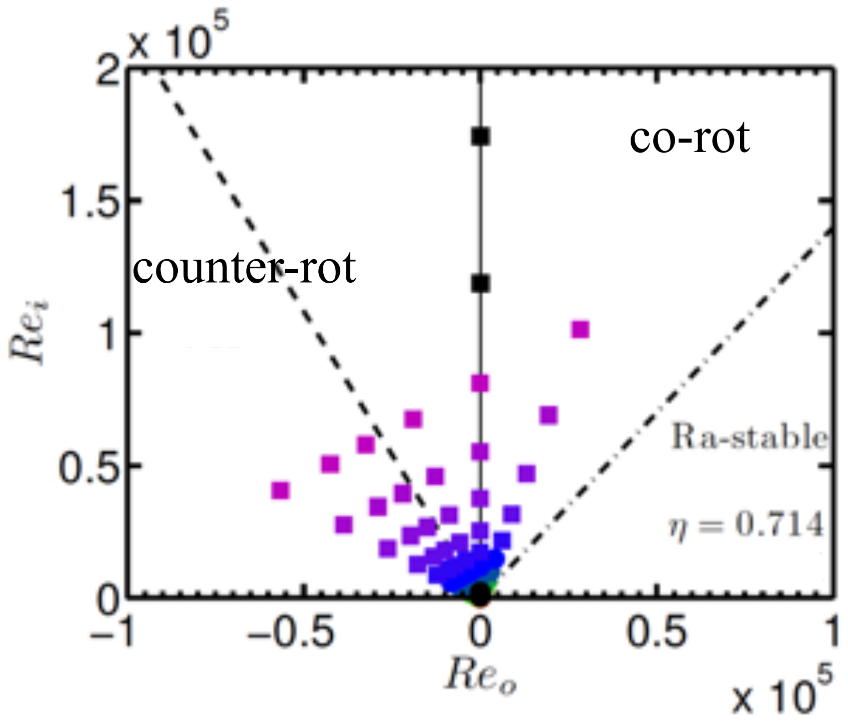
$$Ta = \frac{1}{4}\sigma (r_o - r_i)^2 (r_i + r_o)^2 (\omega_i - \omega_o)^2 \nu^{-2} \propto Re_s^2$$

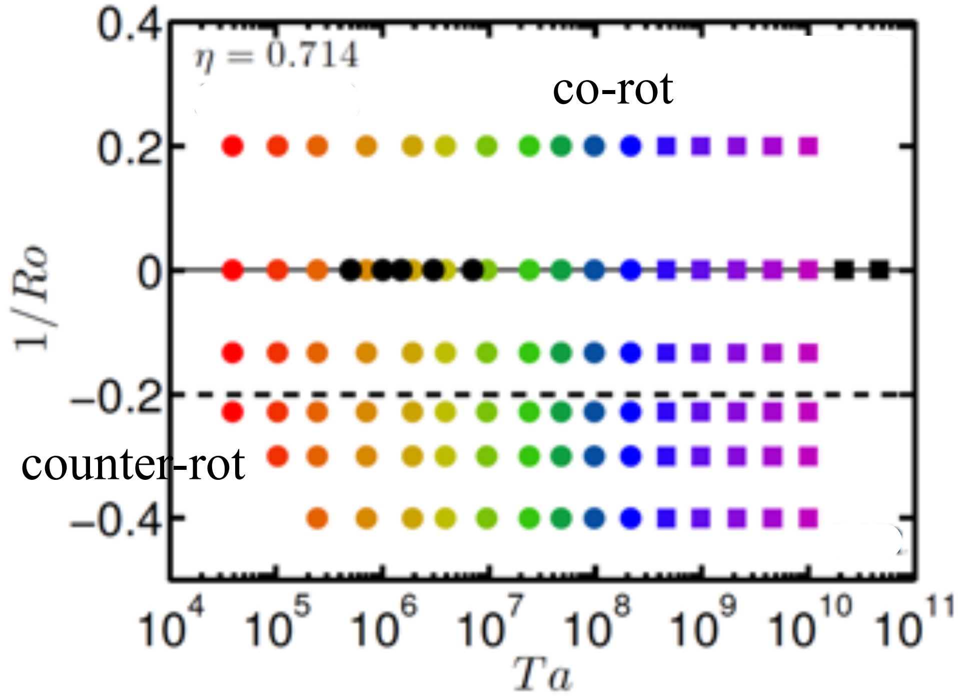
This suggests as control parameters:



(fixed η)

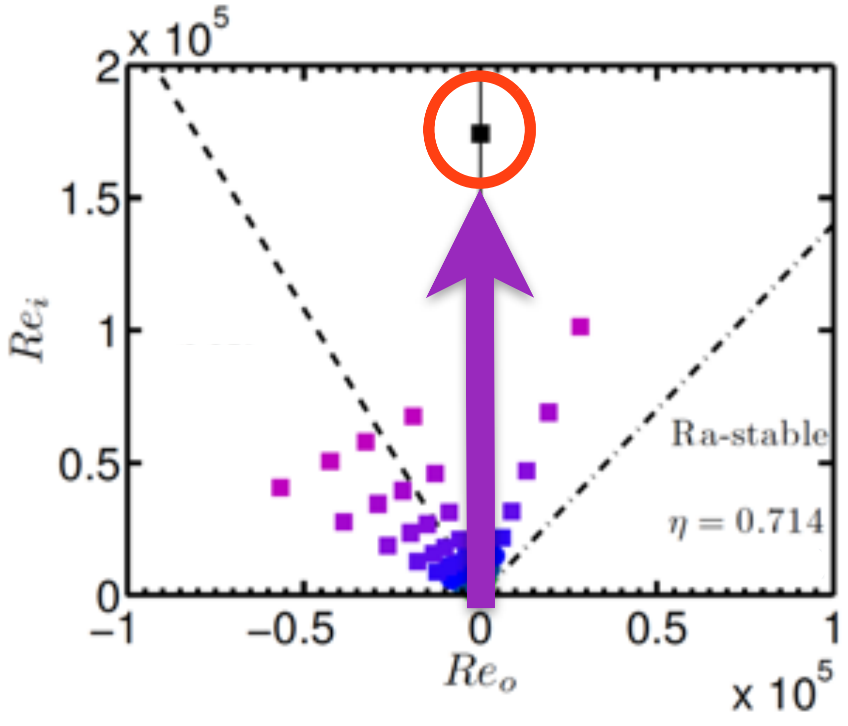
Explore control parameters numerically

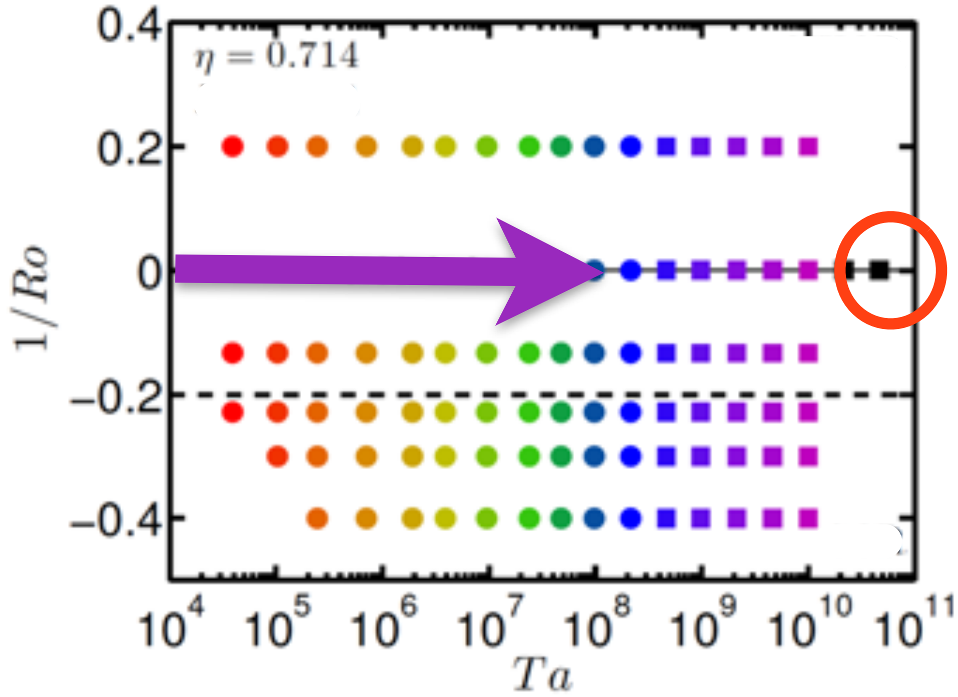




thanks to Prace project ...

First: Fixed outer cylinder:
$Ro^{-1}=0$ or $Re_o=0$
Then increase Ta or Re_i





Global flow properties: Angular velocity transfer Nu_ω

Conserved: angular velocity flux

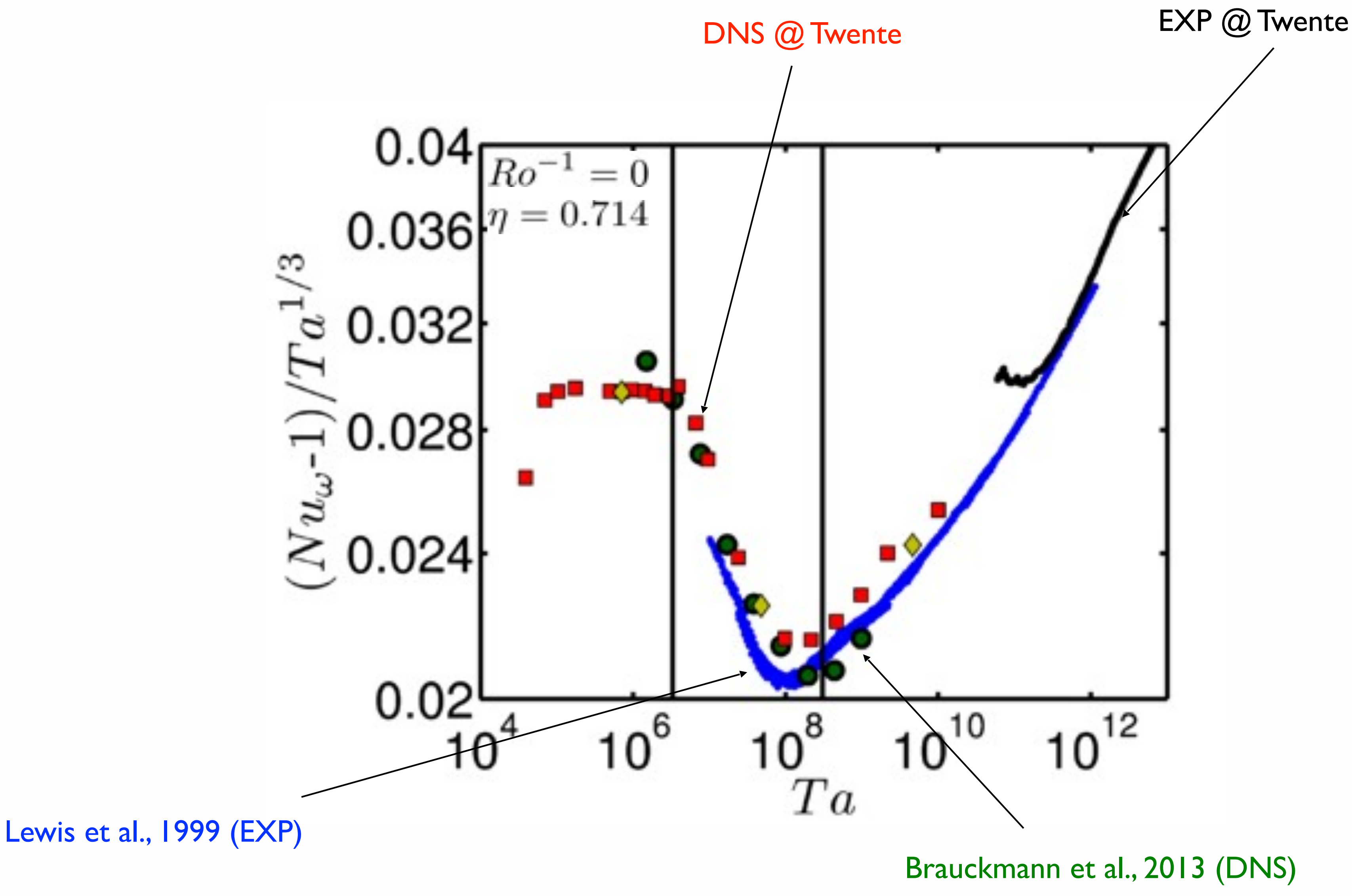
$$J_\omega = r^3 \left[\langle u_r \omega \rangle_{A,t} - \nu \partial_r \langle \omega \rangle_{A,t} \right]$$

$$Nu_\omega = J_\omega / J_{\omega,lam}$$

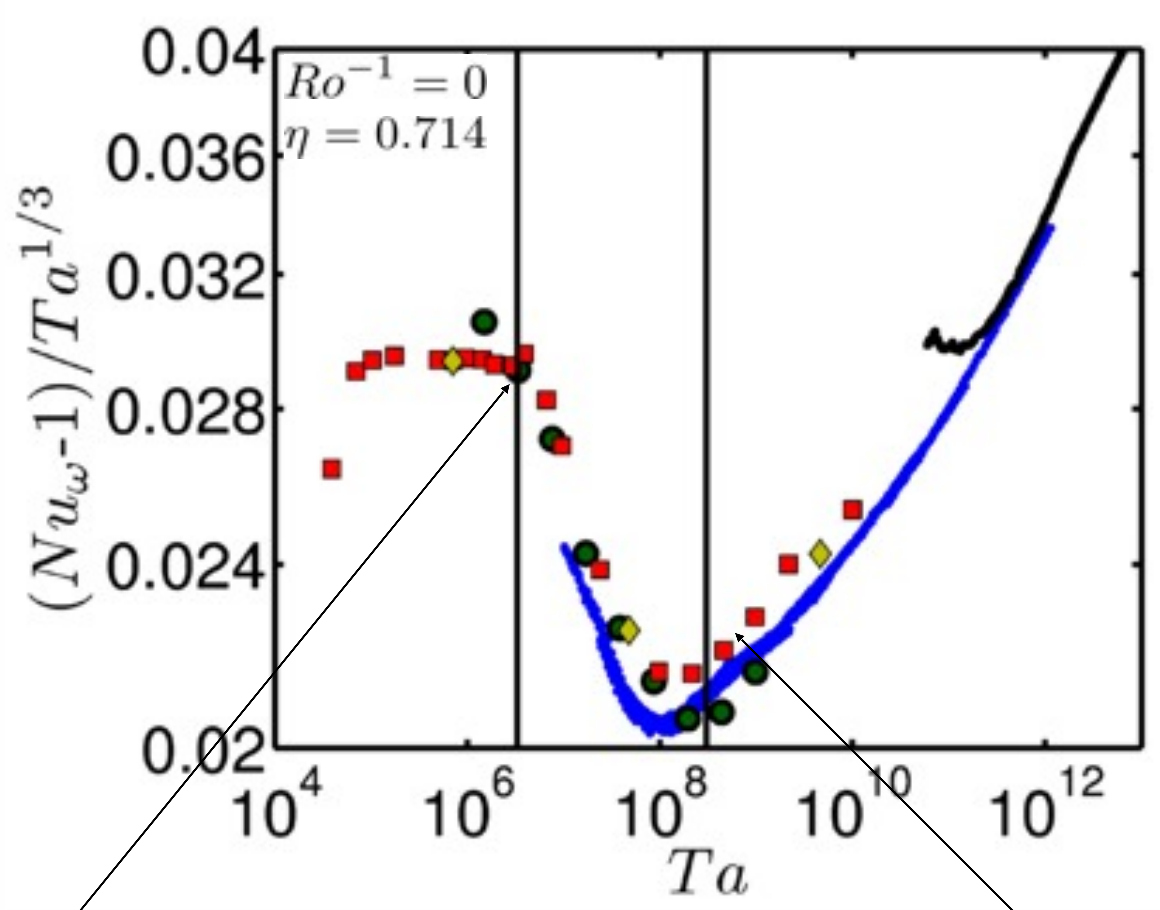
B. Eckhardt, S. Grossmann, and D. Lohse, J. Fluid Mech. 581, 221 (2007)

RB	TC
Conserved: heat flux	Conserved: angular velocity flux
$J = \langle u_z \theta \rangle_{A,t} - \kappa \partial_z \langle \theta \rangle_{A,t}$	$J_\omega = r^3 \left[\langle u_r \omega \rangle_{A,t} - \nu \partial_r \langle \omega \rangle_{A,t} \right]$
$Nu = J / J_{conductive}$	$Nu_\omega = J_\omega / J_{\omega,lam}$
Driven by:	Driven by:
$Ra = \frac{\beta g \Delta L^3}{\kappa \nu}$	$Ta = \frac{d^2 r_a^6}{r_g^4} \frac{(\omega_1 - \omega_2)^2}{\nu^2}$
Scaling: $Nu \propto Ra^\beta$	Scaling: $Nu_\omega \propto Ta^\beta$

Several transitions in Nu_ω



Several transitions in Nu_ω



Ta~2·10^8 : **transition to ultimate regime!**

Ta~3·10^6: very sharp transition

Transition to ultimate regime = Transition of BL from (laminar) Prandtl-Blasius type to (turbulent) Prandtl-von Karman type

Prandtl

Blasius

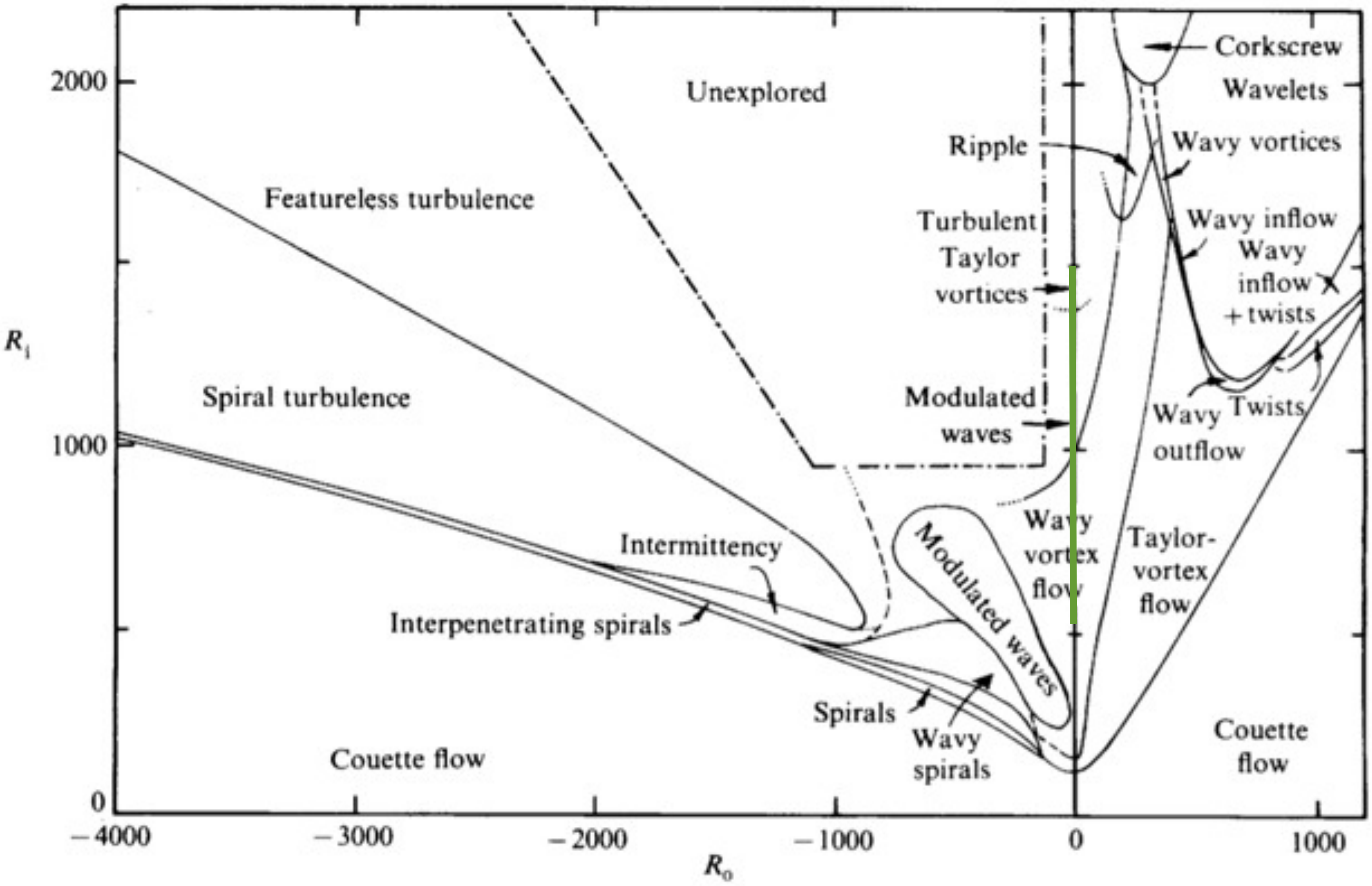
$Ta = 5 \times 10^8$

Prandtl

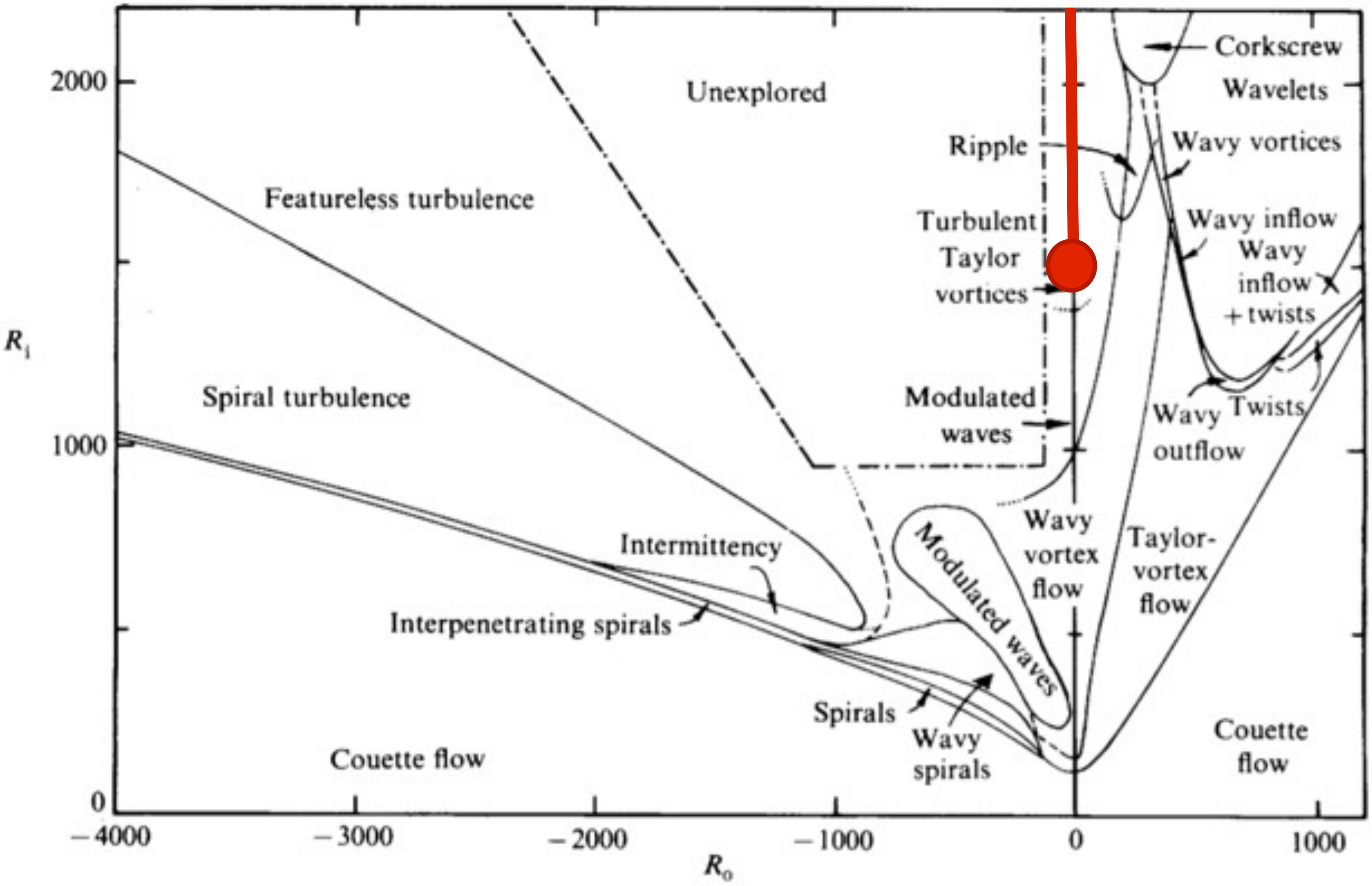
von Karman

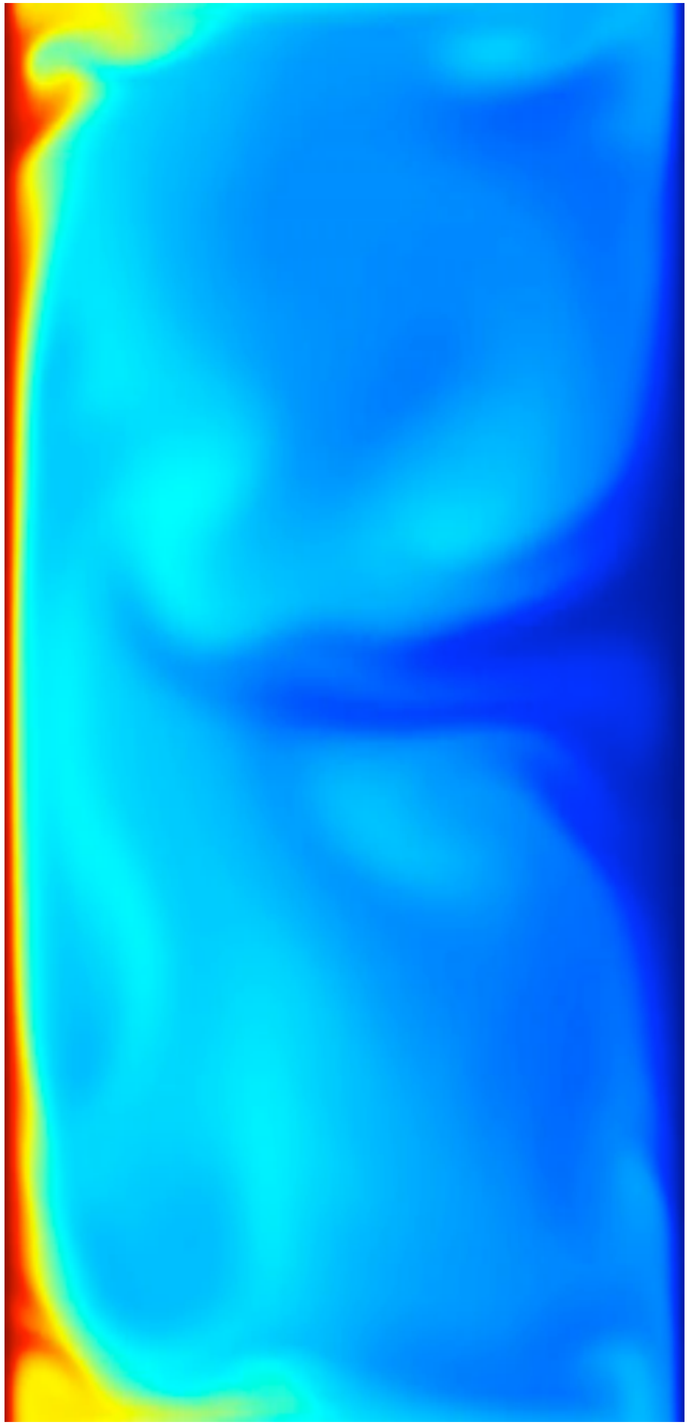
How do the corresponding flow patterns look like?

Small Ta: laminar Taylor rolls (time dependent)



Transition to turbulent Taylor vortices at Ta ~ 3·10^6



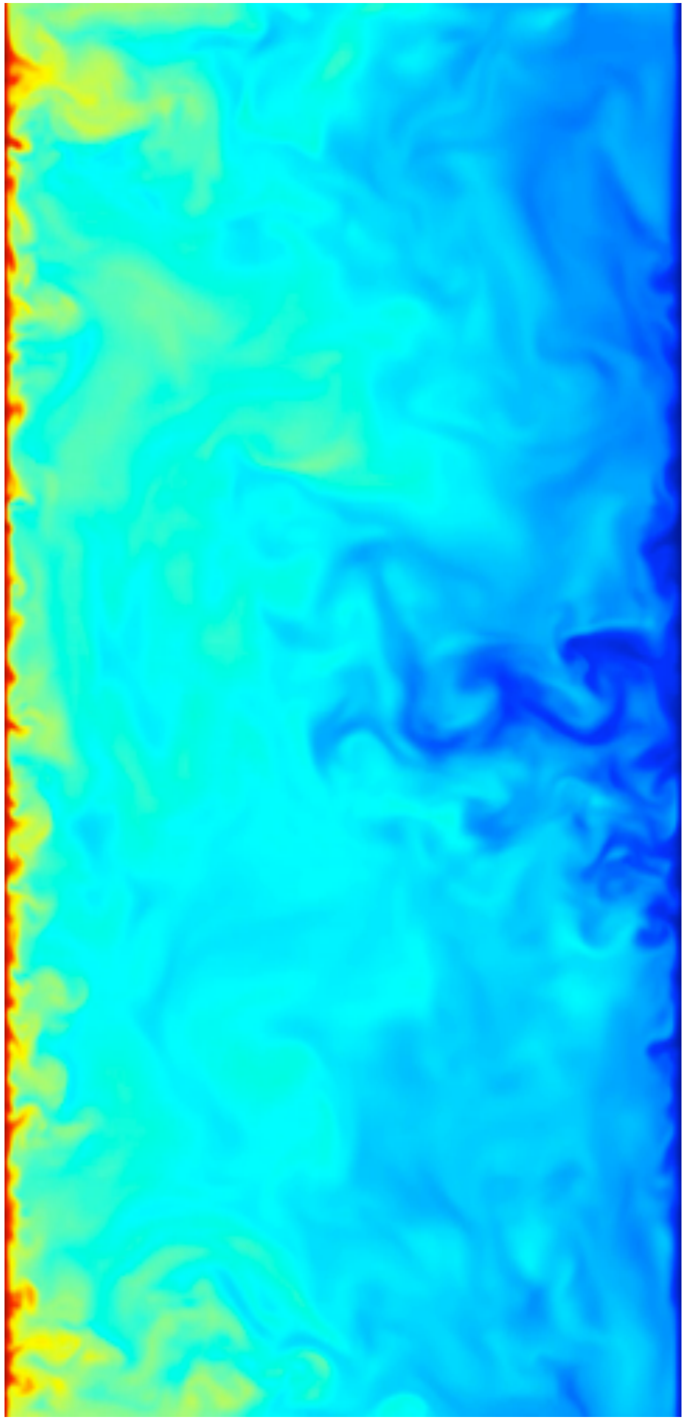


Turbulent Taylor vortices, but laminar-type BL

$Ta = 5 \times 10^7$

$Ro^{-1} = 0$

$\eta = 0.714$



Transition to turbulent BL at $Ta = 2 \times 10^8$:
"ultimate turbulence"

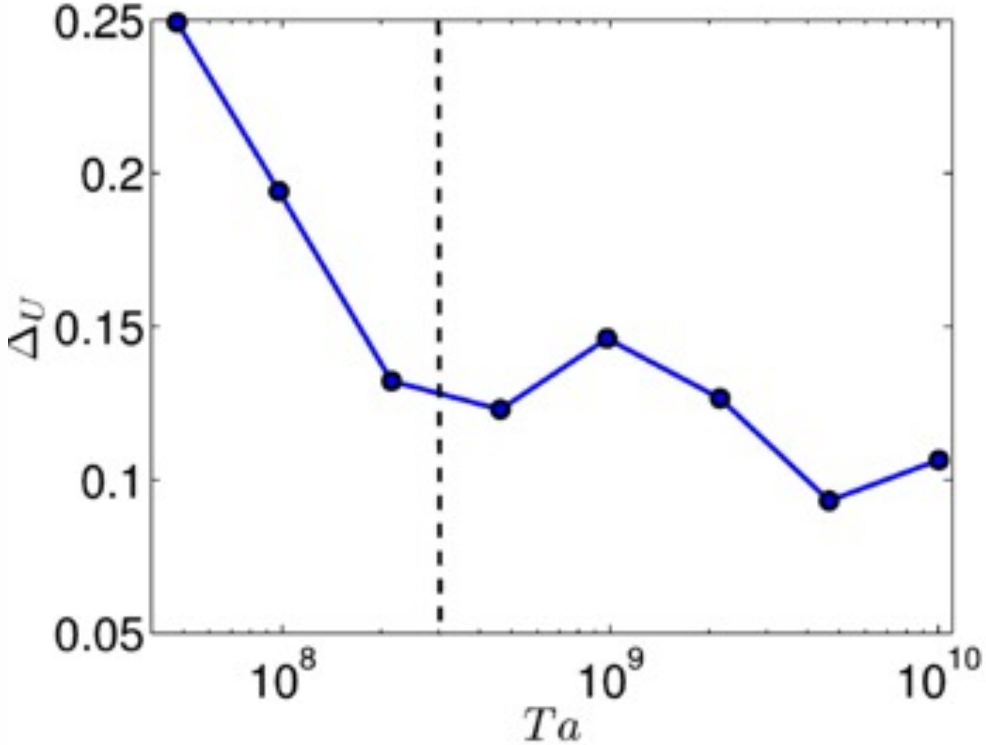
$Ta = 4 \times 10^9$

$Ro^{-1} = 0$

$\eta = 0.714$

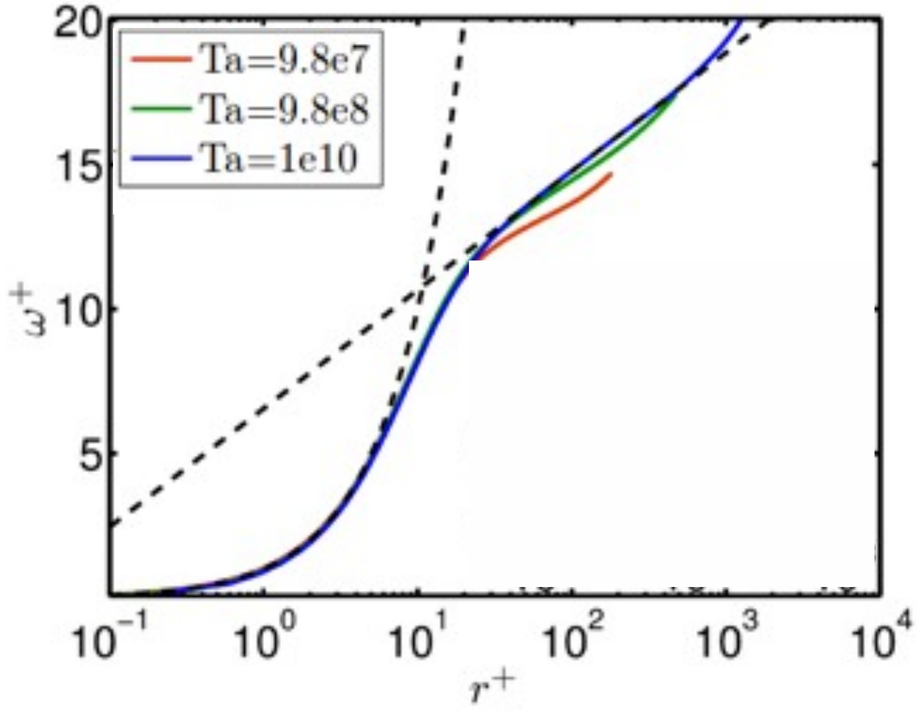
Attempt of quantification: axial velocity spread Δ_U

$$\Delta_U = \frac{\max_z \hat{u}_\theta(r_{1/2}, z) - \min_z \hat{u}_\theta(r_{1/2}, z)}{\langle \hat{u}_\theta(r_{1/2}, z) \rangle_z}$$



Large scale structures become less beyond transition

BL beyond transition to turbulence: Logarithmic profile develops

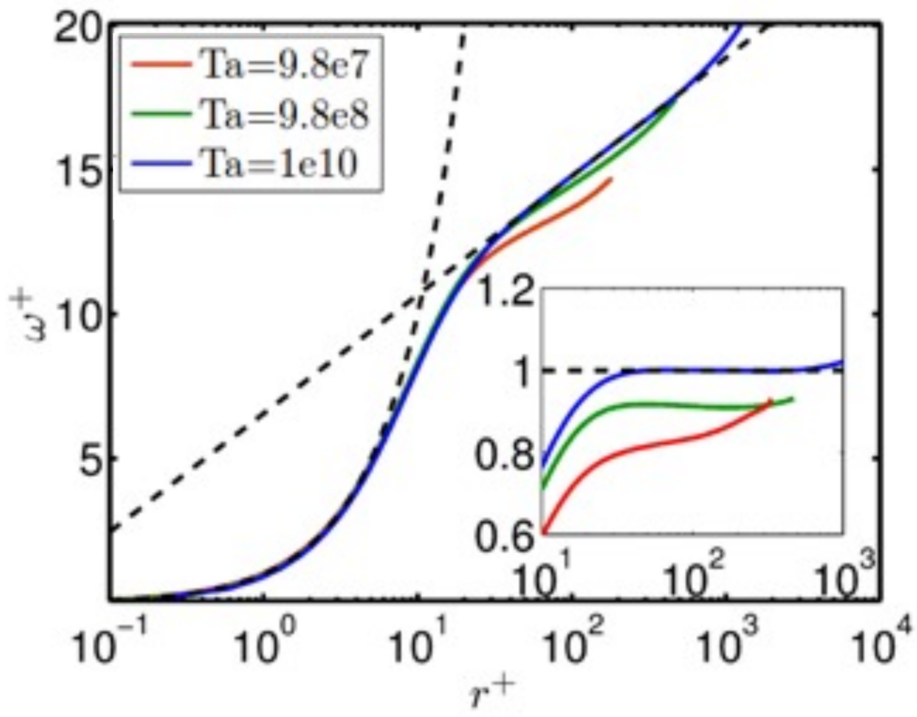


$$\omega^+ = \frac{\omega}{\omega_\tau}$$

$$r^+ = \frac{r - r_i}{\delta_\nu}$$

<u>NOT</u> the azimuthal velocity $u_{\theta,}$ but the angular velocity ω due to the cylindrical shape

BL beyond transition to turbulence: Logarithmic profile develops

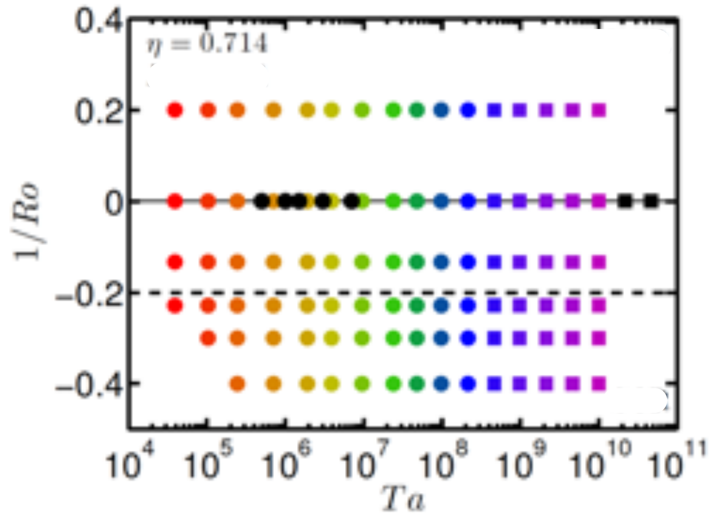


$$\omega^+ = \frac{\omega}{\omega_\tau}$$

$$r^+ = \frac{r - r_i}{\delta_\nu}$$

<u>NOT</u> the azimuthal velocity u_{θ}, but the angular velocity ω due to the cylindrical shape

How do transitions change with outer cylinder rotation $Ro^{-1} \neq 0$?



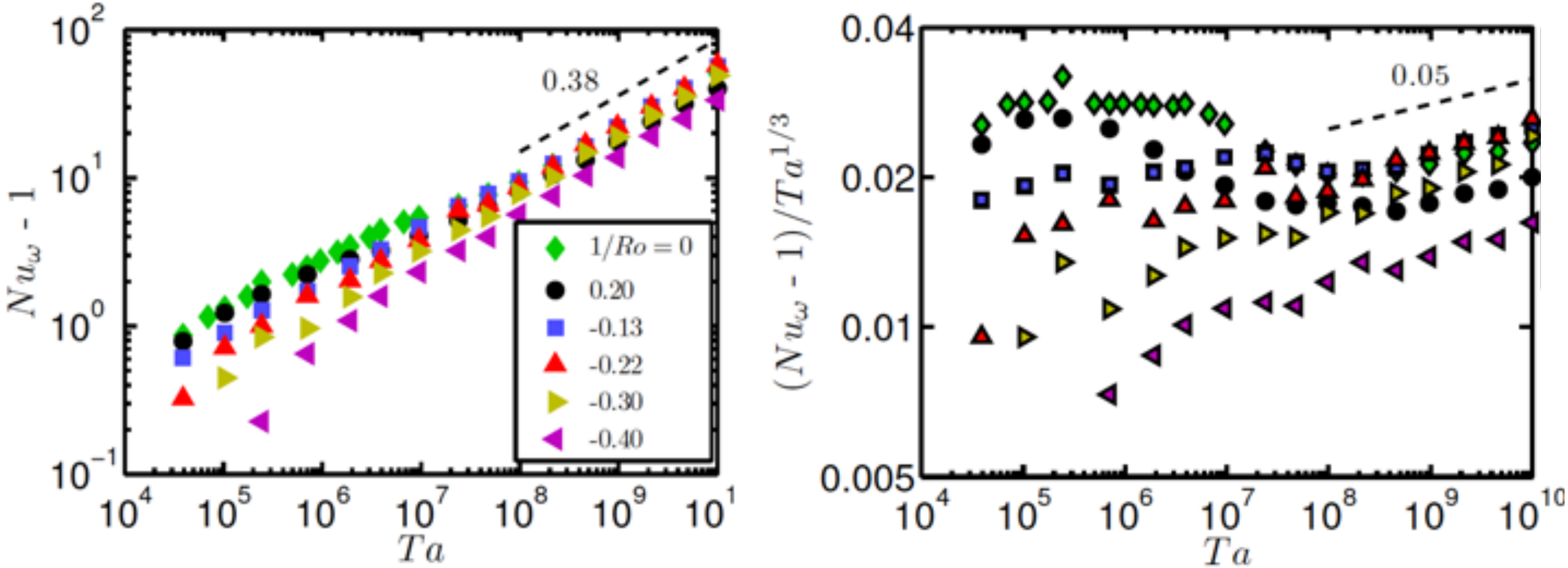
Expectation:

Co-rotation $Re_o > 0$ or $Ro^{-1} > 0$:
flow stabilization

Weak counter-rotation $Re_o < 0$ or $Ro^{-1} < 0$:
flow destabilization!

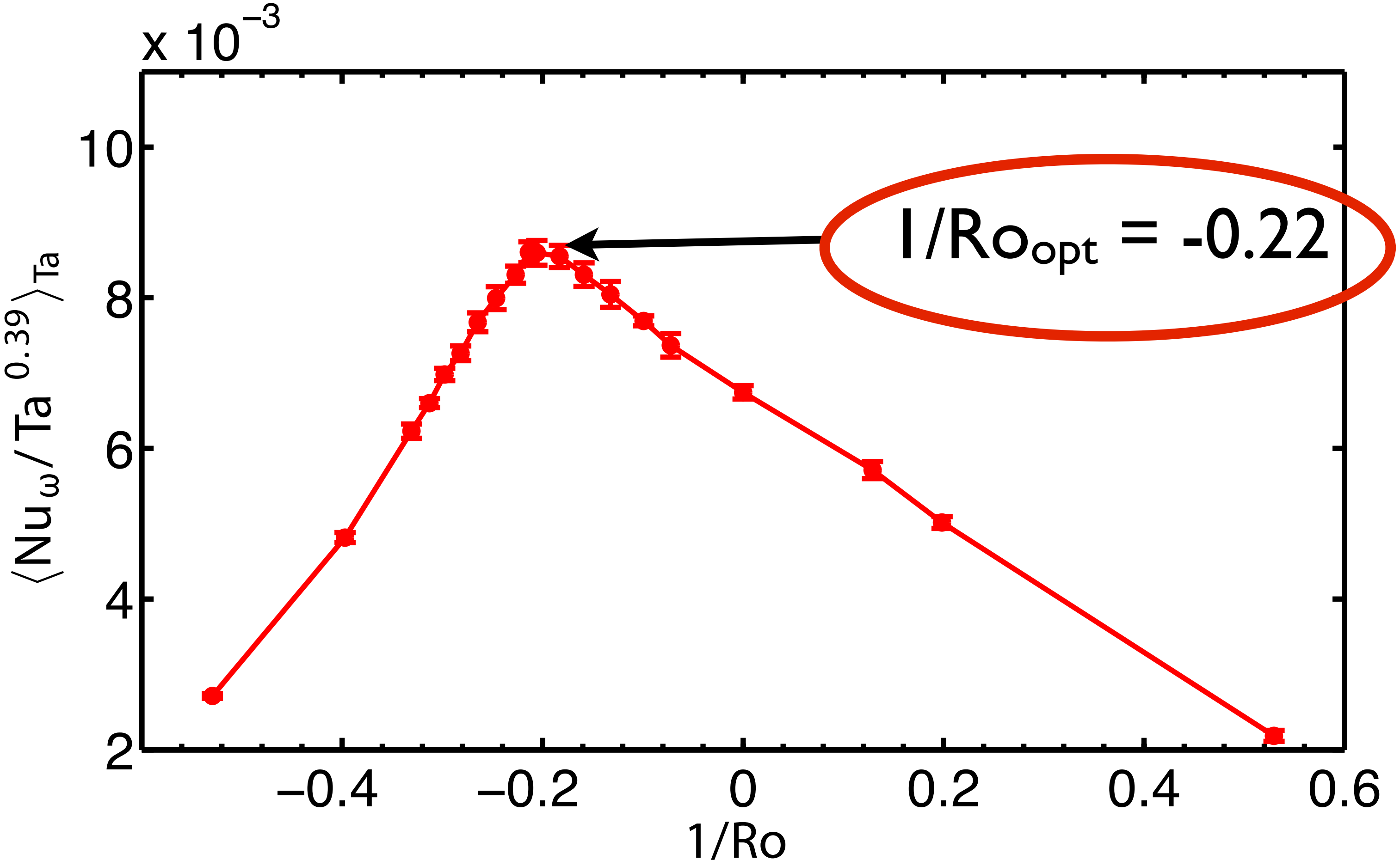
Strong counter-rotation $Re_o << -|Re_o| < 0$ or $Ro^{-1} << 0$:
flow stabilization again

Global flow properties: Nuω



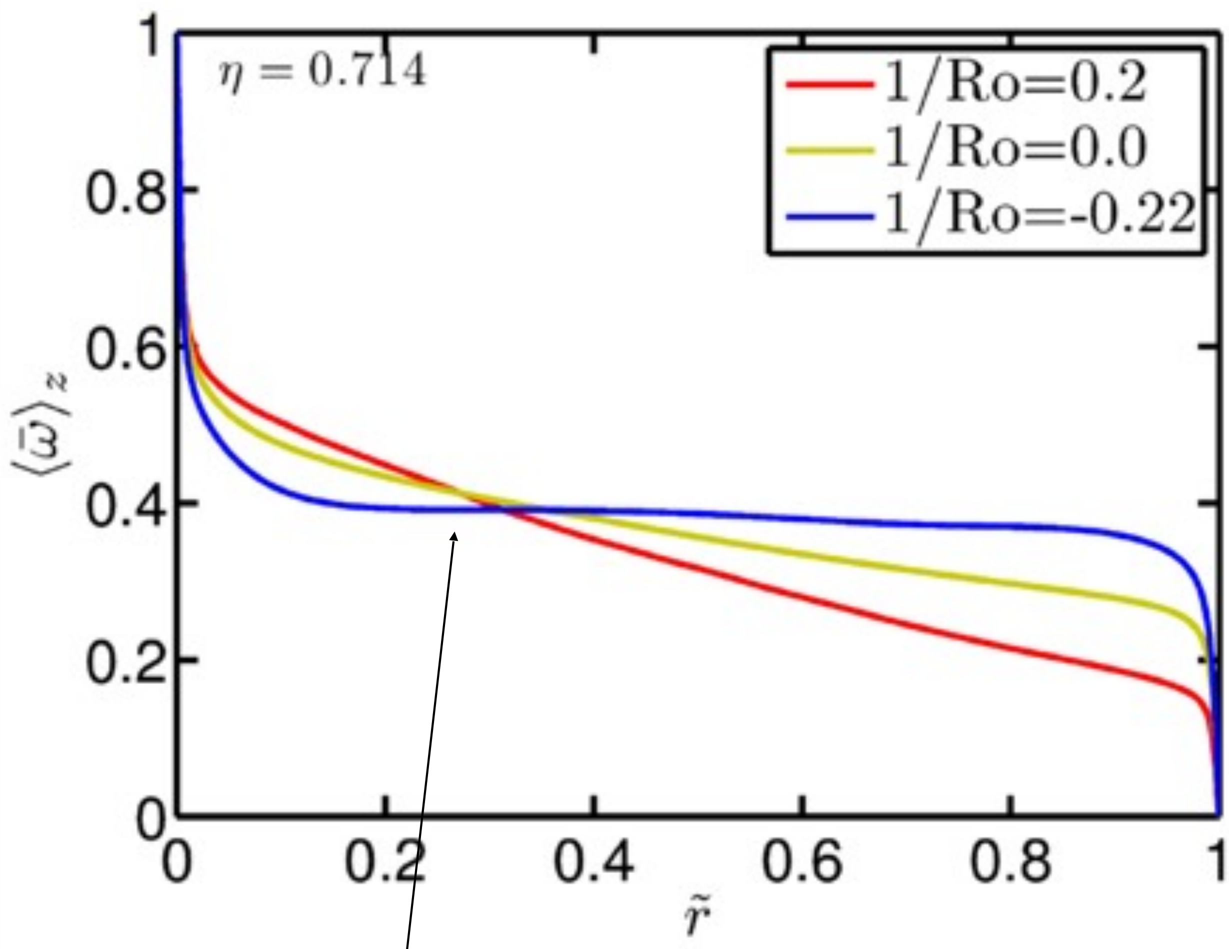
Transition to ultimate regime looks universal

Global flow properties: Nu_ω (Ro^{-1}) and "optimal" TC turbulence



R. Ostilla-Monica et al., JFM 747, 1-29 (2014)

Local flow organization: ω-profiles



co-rotation

counter-rotation, close to optimal TC

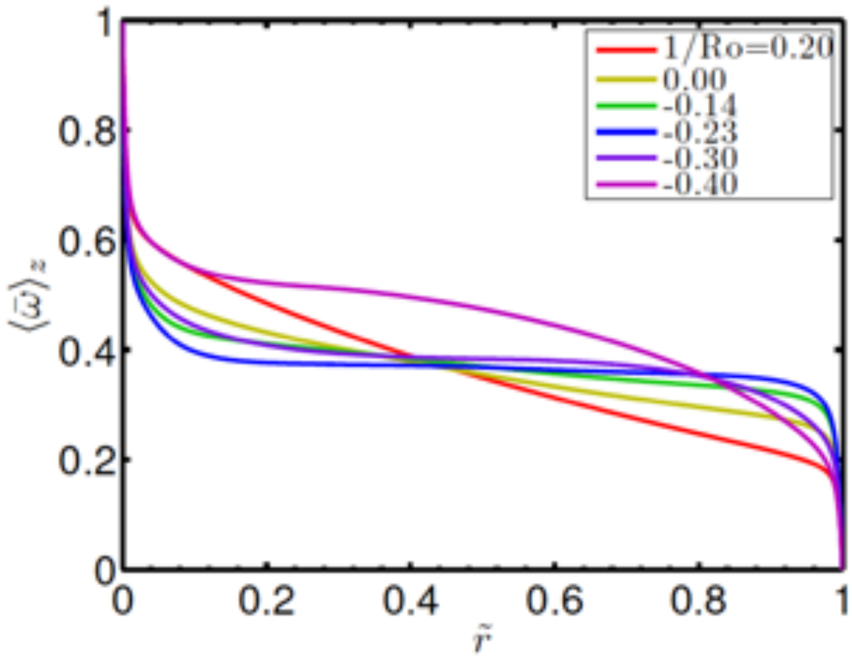
Gradient in the bulk means that plumes enter bulk with lower energy

Flat bulk means plumes enter bulk with high energy

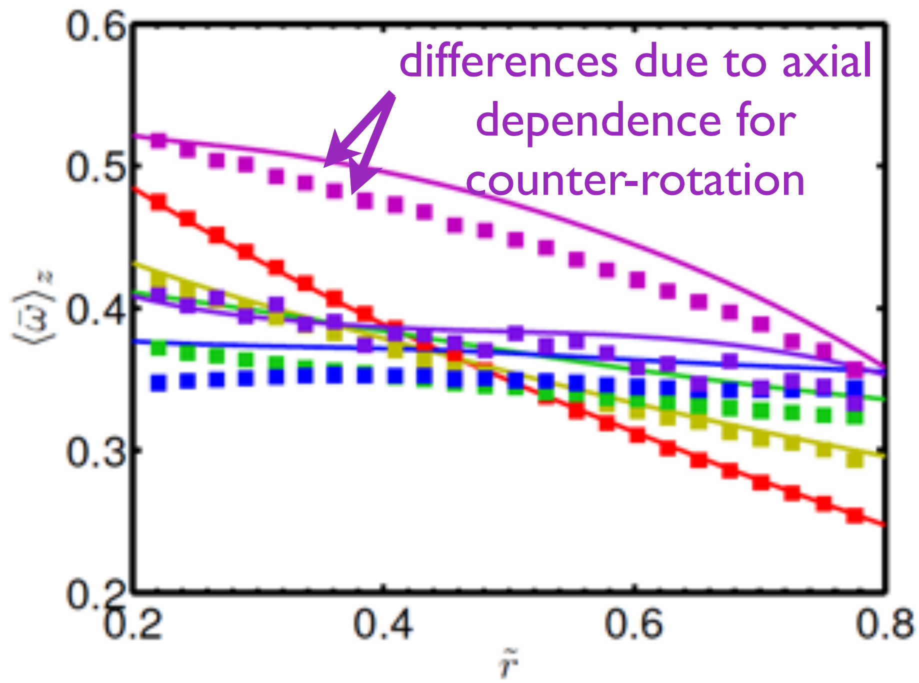
$\eta = 0.714$, $Ta = 10^{10}$

Local flow organization: ω-profiles

$\eta = 0.714$, $Ta = 10^{10}$



Comparison with experiment:



$Ro^{-1} = 0.2$

co-rotation: gradient in center due to good plume mixing

$Ro^{-1} = -0.23$

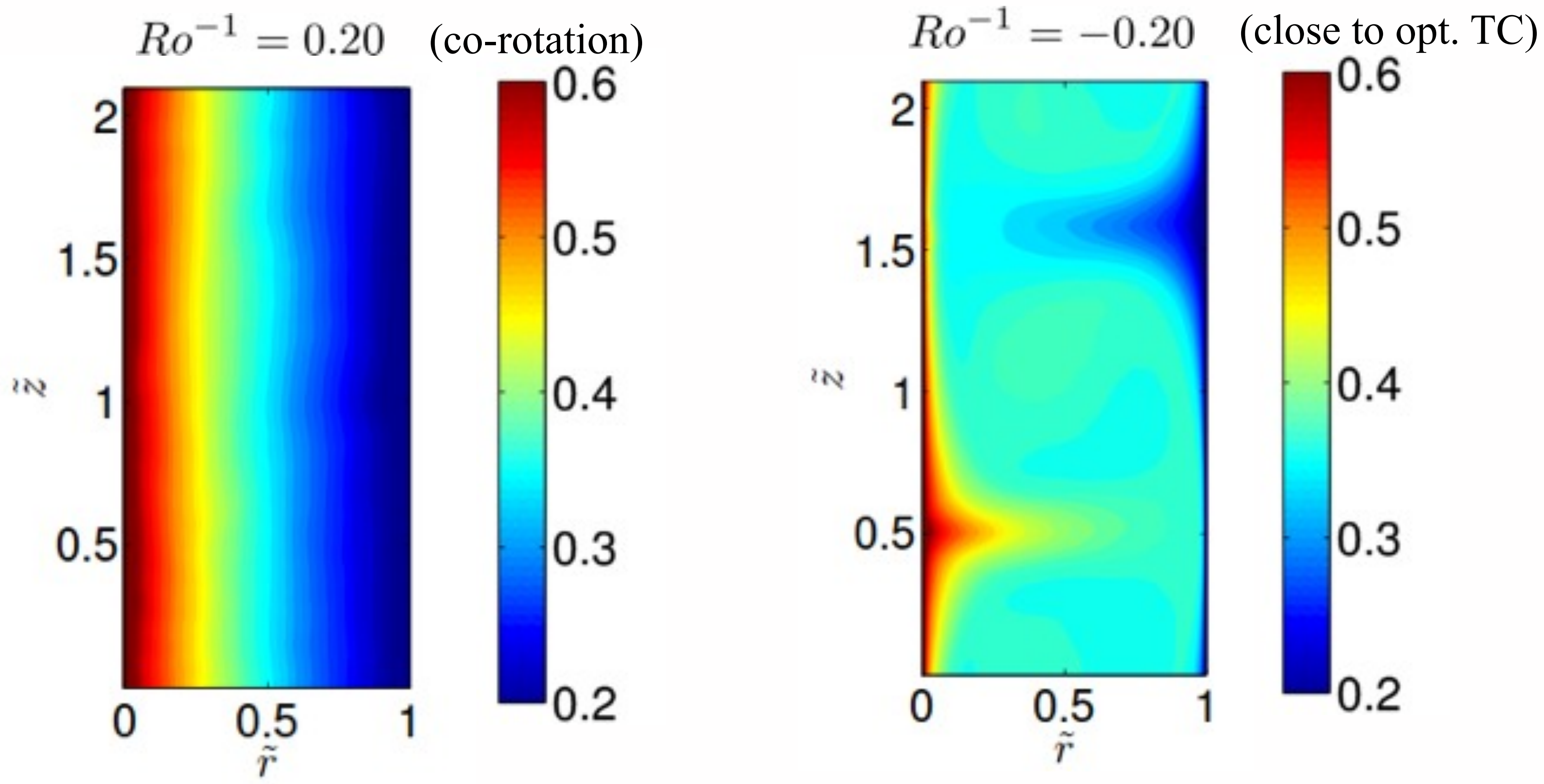
"Optimal TC": flat profile; less plume mixing

$Ro^{-1} = -0.40$

Ra-stable zone at outer cylinder

Local flow organization: $\langle\omega\rangle_{t,\theta}$

Stronger plumes imply: rolls survive at higher Ta

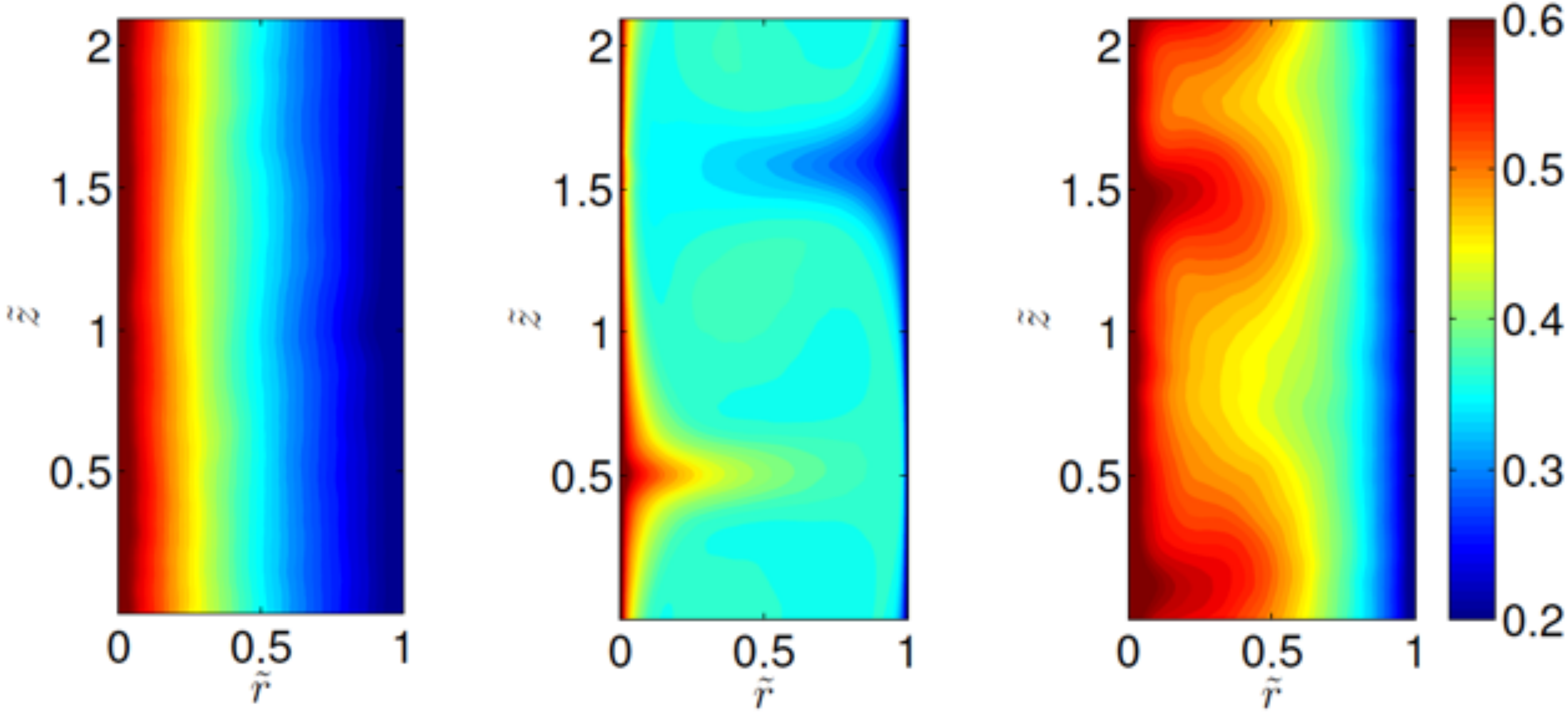


Rolls = more convective transport = transport optimum!

$Ta = 10^{10}$

Local flow organization: $\langle\omega\rangle_{t,\theta}$

$\eta = 0.714$, $Ta = 10^{10}$



$Ro^{-1} = 0.2$

co-rotation: no axial dependence: weak plumes, good mixing

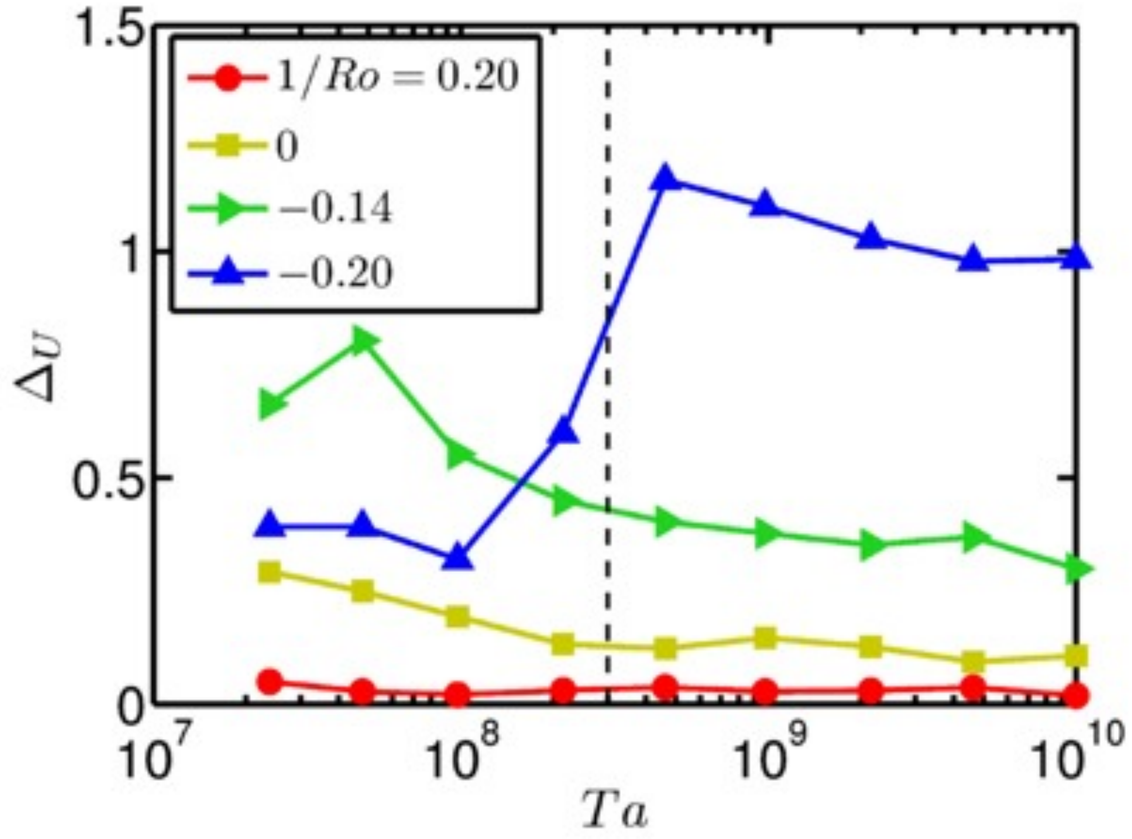
$Ro^{-1} = -0.22$

"Optimal TC": axial depence, rolls; strong plumes

$Ro^{-1} = -0.40$

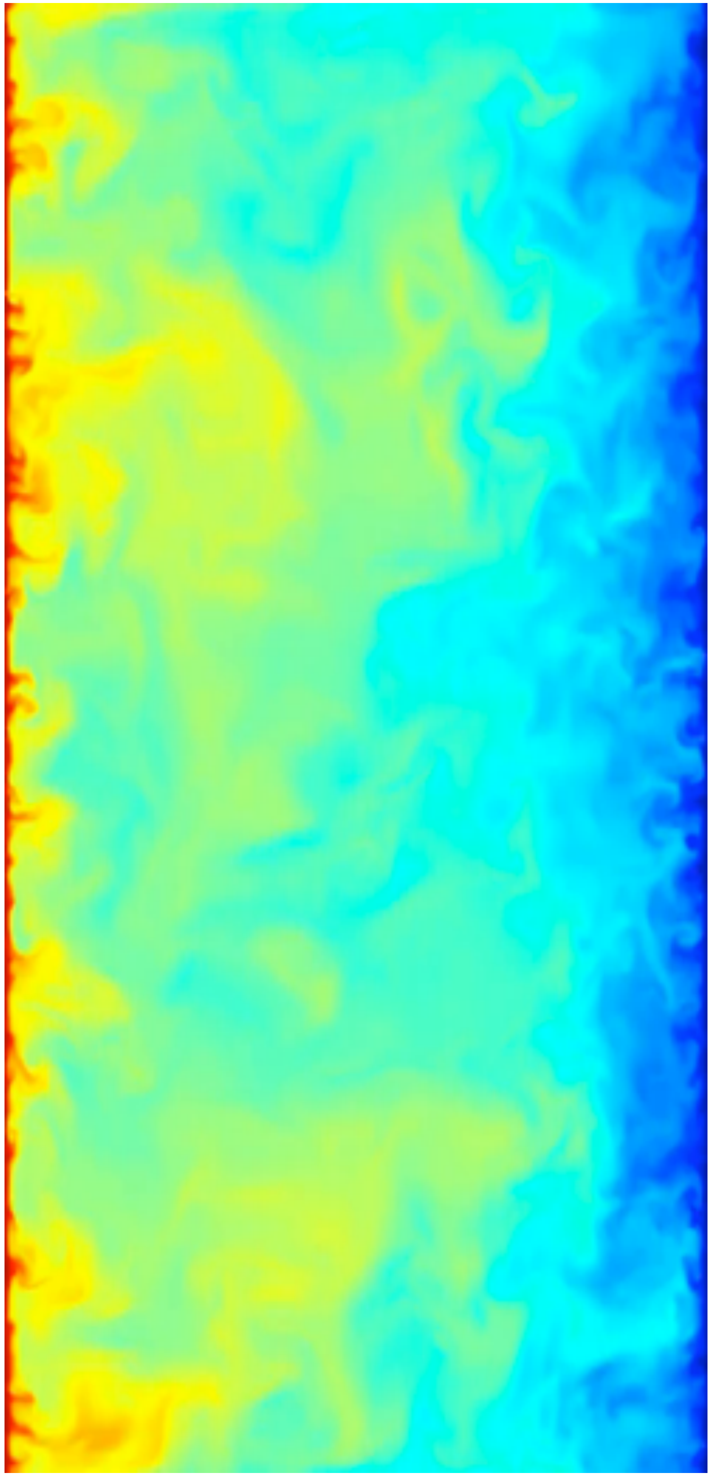
Rolls at inner cylinder, but Ra-stable zone at outer cylinder

Quantification of roll structure by axial velocity spread Δ_U



Good transport at optimal TC due to the surviving roll structure

Largest Δ_U in ultimate regime at optimum Ro^{-1}!



Flow features in strongly counter-rotating regime

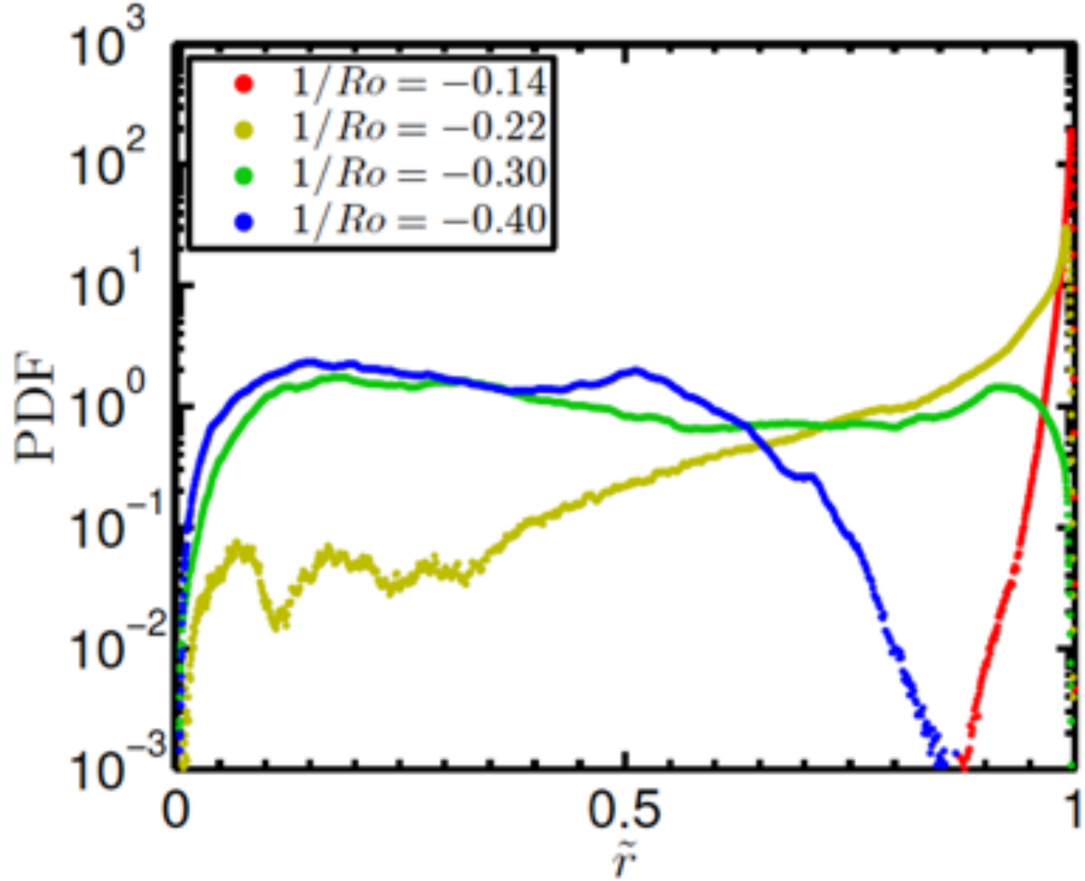
$Ta = 4 \times 10^9$

$Ro^{-1} = -0.4$

$\eta = 0.714$

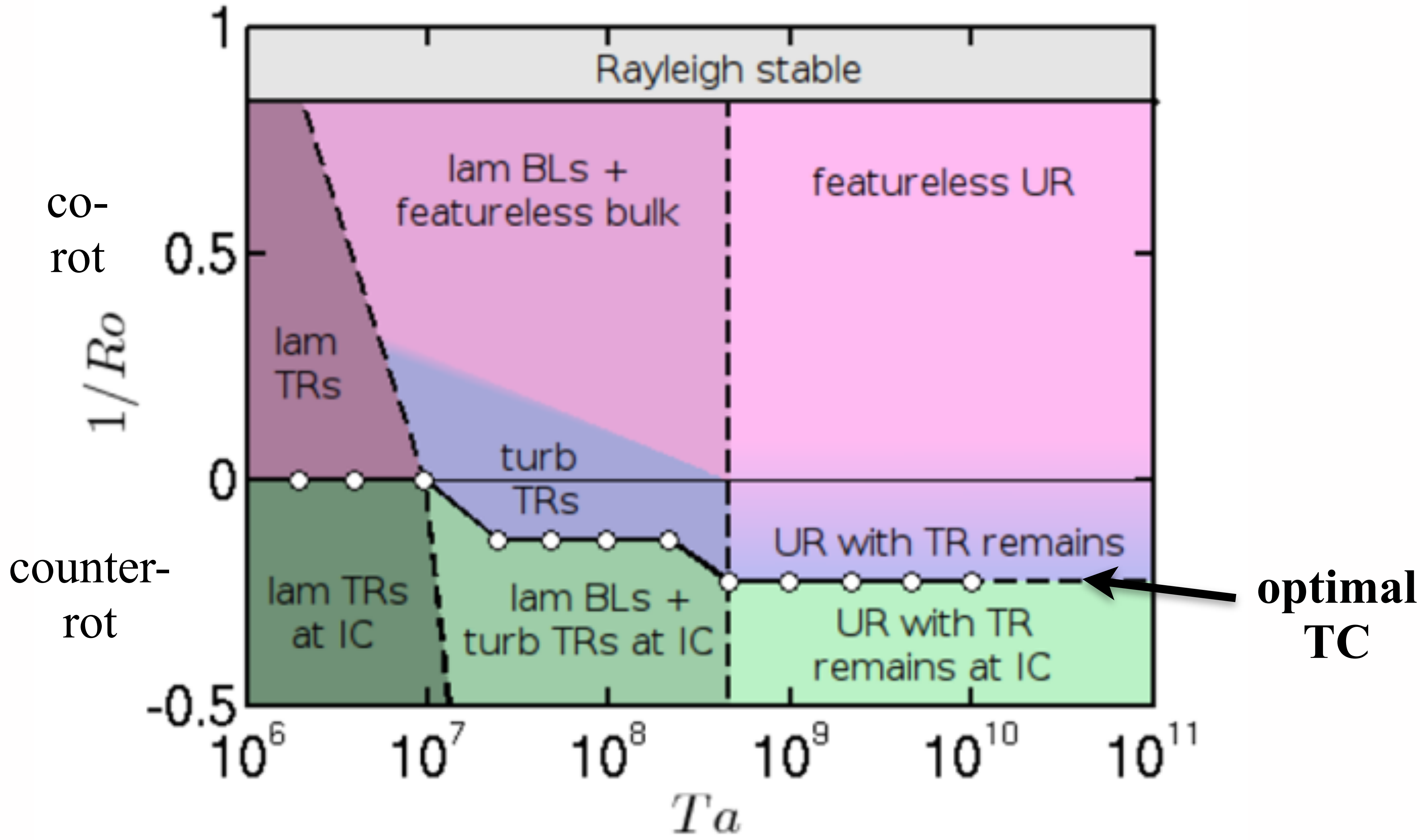
Position of neutral surface $<\omega(r)>_{t,\theta,z}$ =0

$\eta = 0.714$, $Ta = 10^{10}$

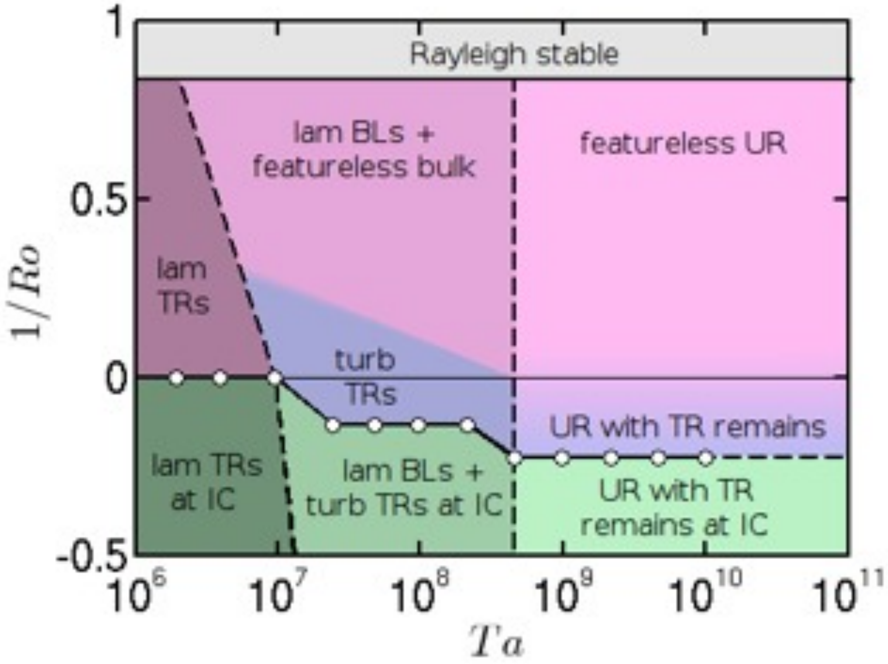


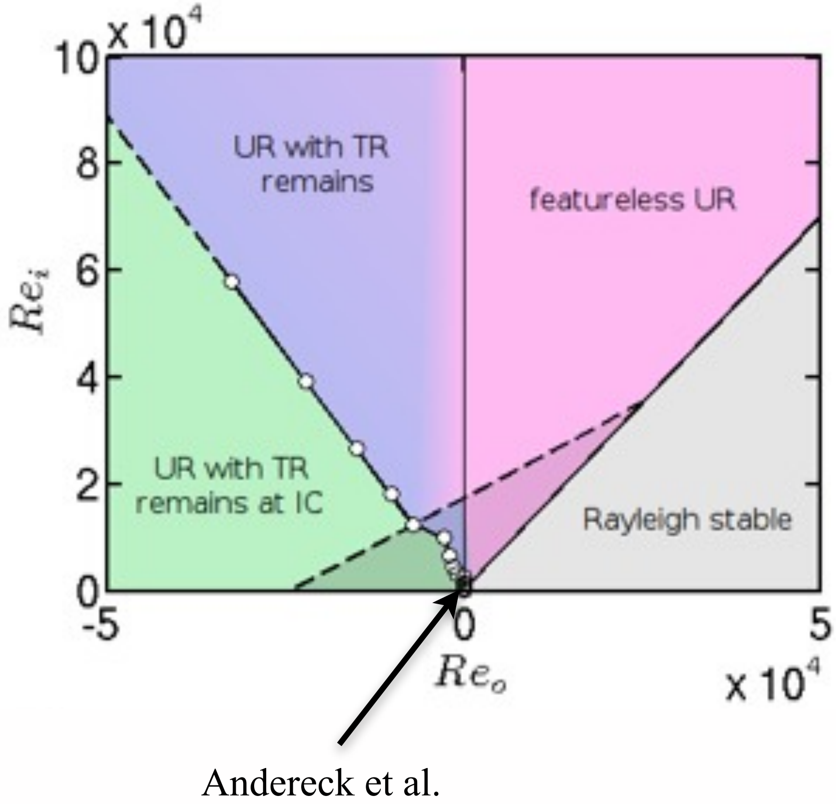
Neutral line migrates inwards with increasing counter-rotation

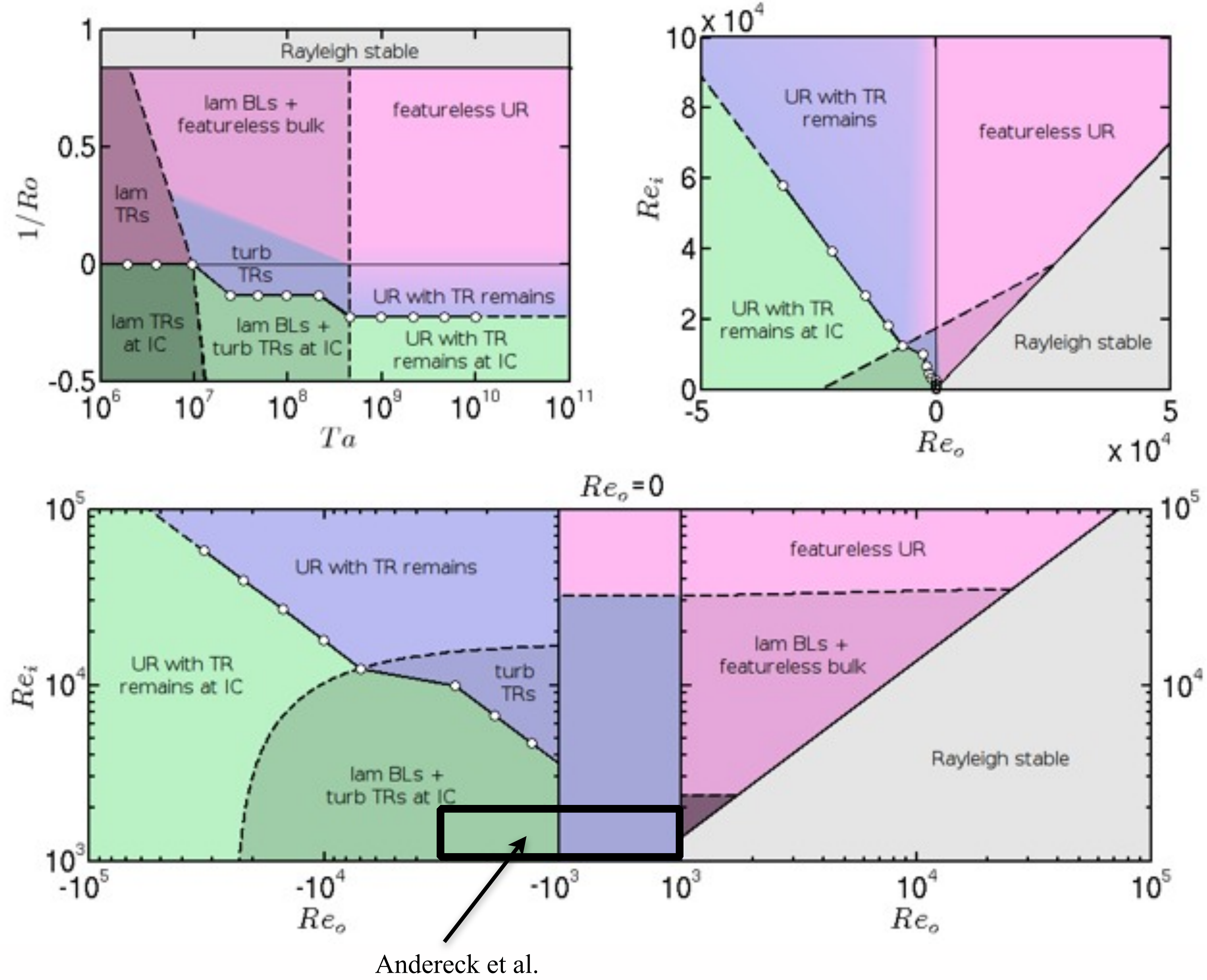
Summary: Phase diagram



Summary: Phase diagram

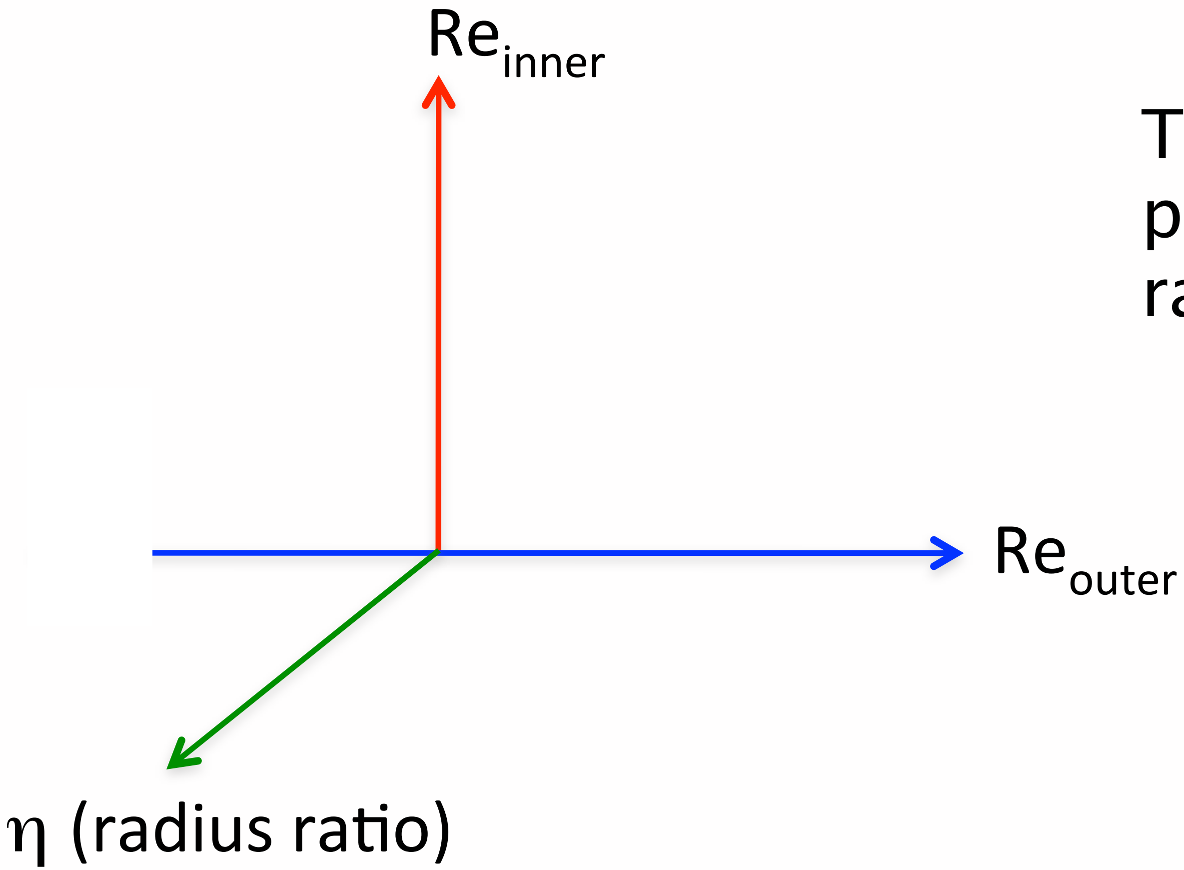






Rayleigh stable
lam BLs + featureless bulk
featureless UR
lam TRs
turb TRs
UR with TR remains
lam TRs at IC
lam BLs + turb TRs at IC
UR with TR remains at IC
1/Ro
Ta
UR with TR remains
featureless UR
UR with TR remains at IC
Rayleigh stable
Re_i
Re_o
x 10^4
Re_o=0
UR with TR remains
featureless UR
UR with TR remains at IC
turb TRs
lam BLs + featureless bulk
lam BLs + turb TRs at IC
Rayleigh stable
Re_i
Re_o
Andereck et al.

How does the phase space depend on the radius ratio?



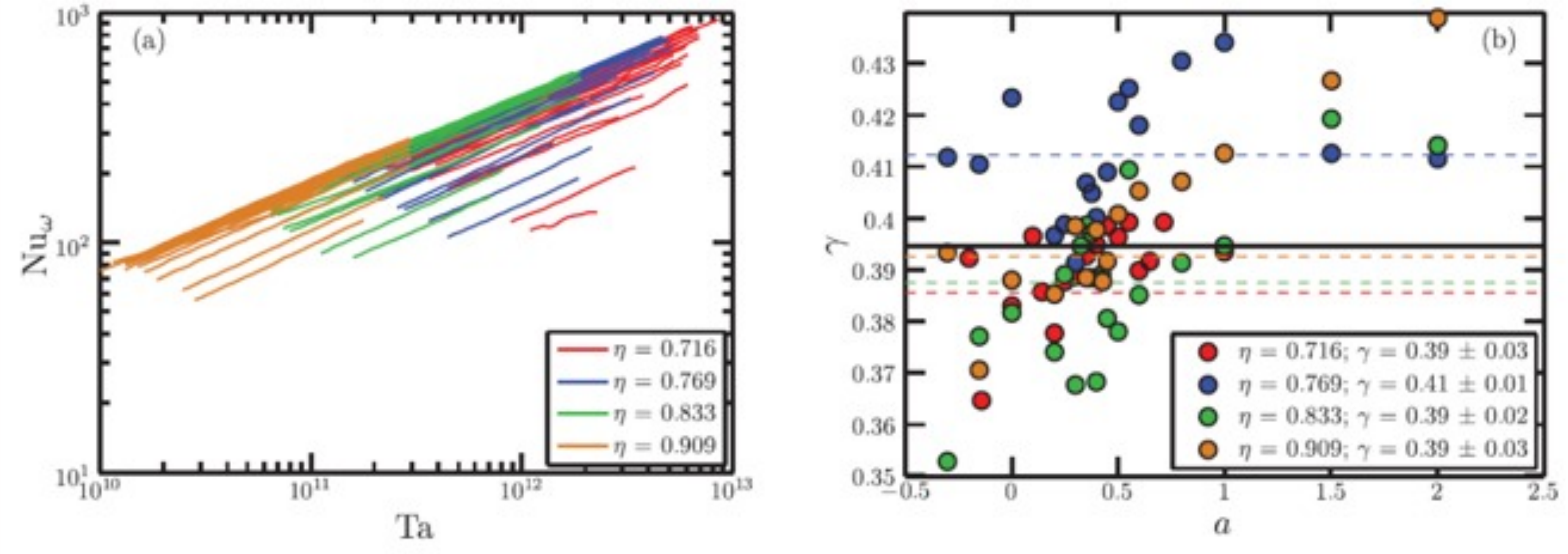
Third axis in parameter space: radius ratio η

$\eta \to 0$ Vanishing inner cylinder

$\eta \to 1$ Two plates (plane Couette)

$$Nu_\omega(\eta, Ro^{-1}, Ta)$$

Can one see universal scaling for other values of η?



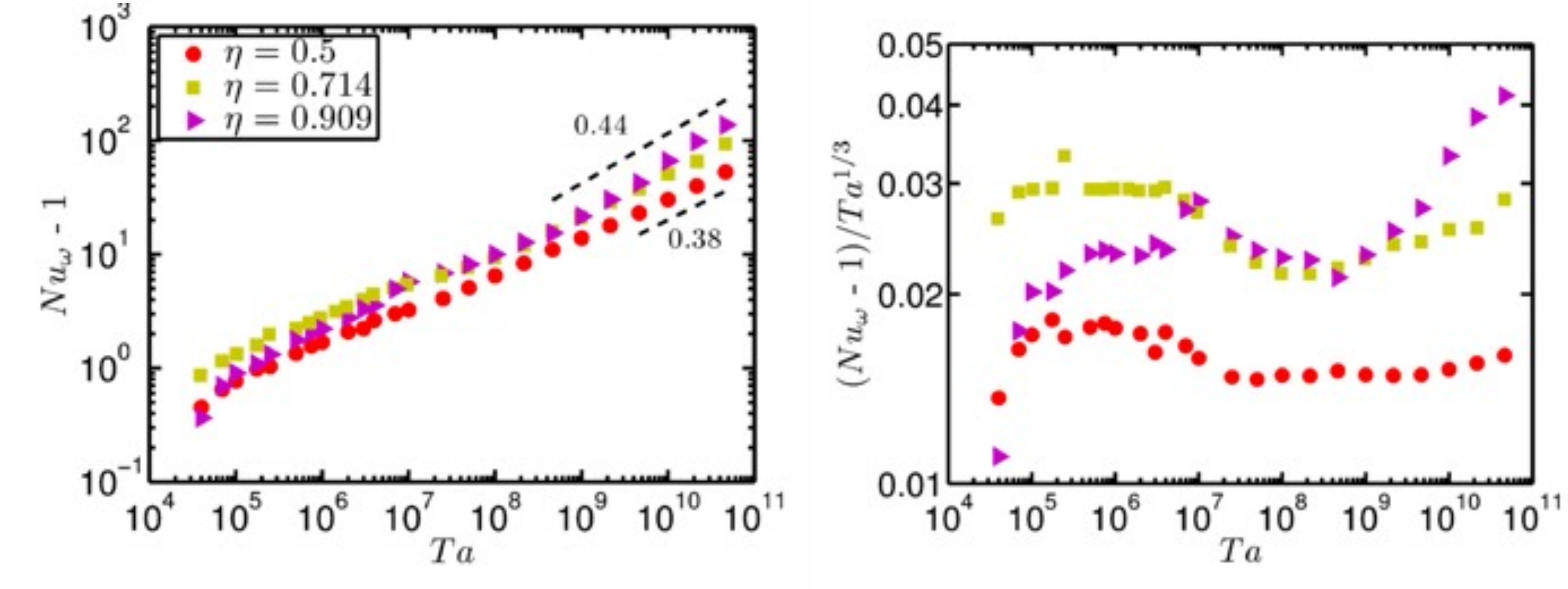
$$\gamma = 0.395 \pm 0.03$$

Ultimate regime scaling is found for all η!

γ = 0.38 also found at η=0.5 by Merbold et. al, PRE (2013)

Ostilla, Huisman, Jannink, van Gils, Verzicco, Grossmann, Sun & Lohse, J. Fluid Mech. 747, 1-29 (2014)

When does the transition happen ?



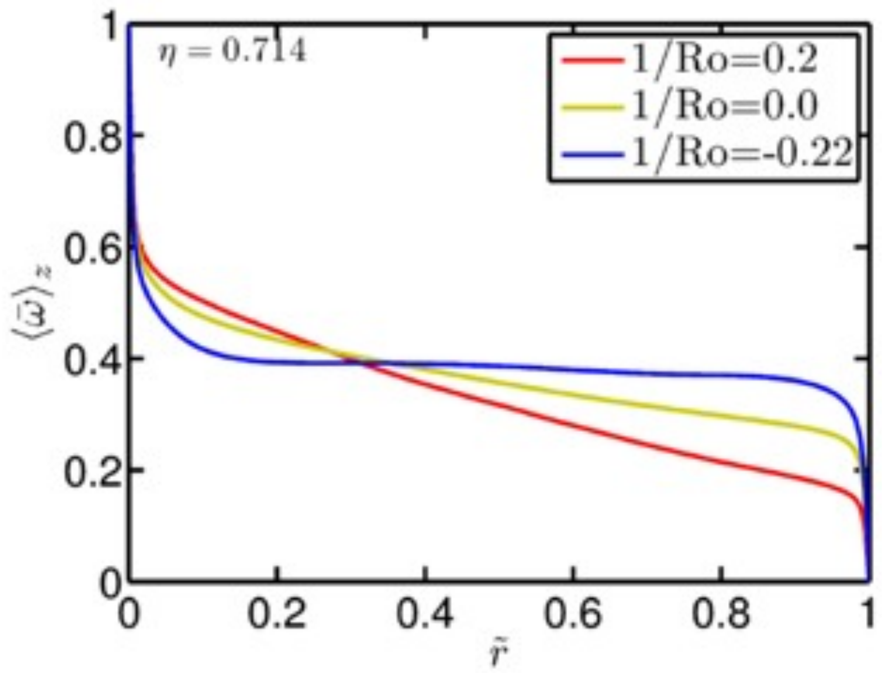
$Ro^{-1} = 0$

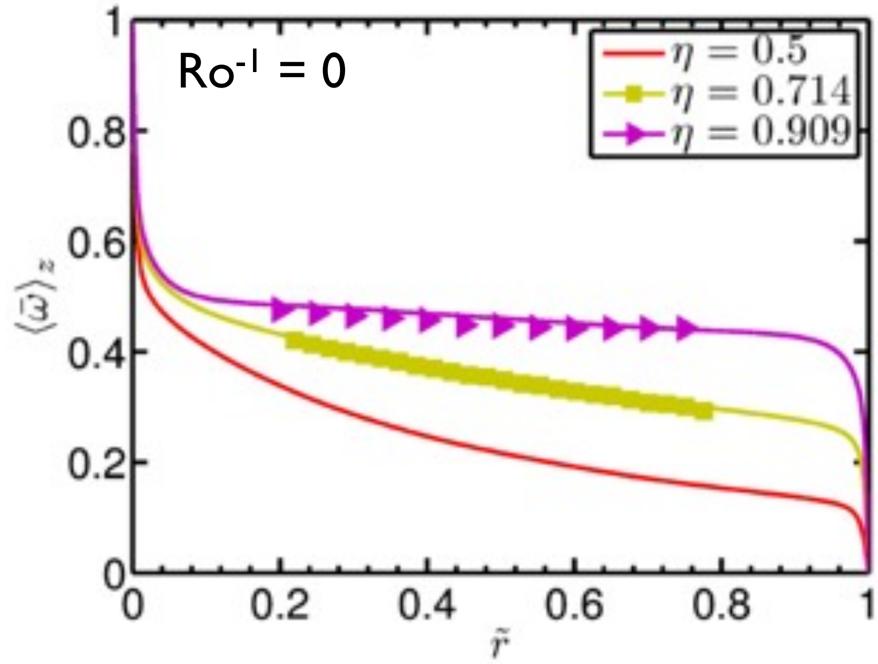
η=0.909, Ta~3·10^{8} – also seen in Ravelet et. al (2011)

η=0.5, Ta~10^{10} – also seen in Merbold et. al (2013)

similar for larger η -- later for smaller η

Analogy between Ro^{-1} and η: effect on profile

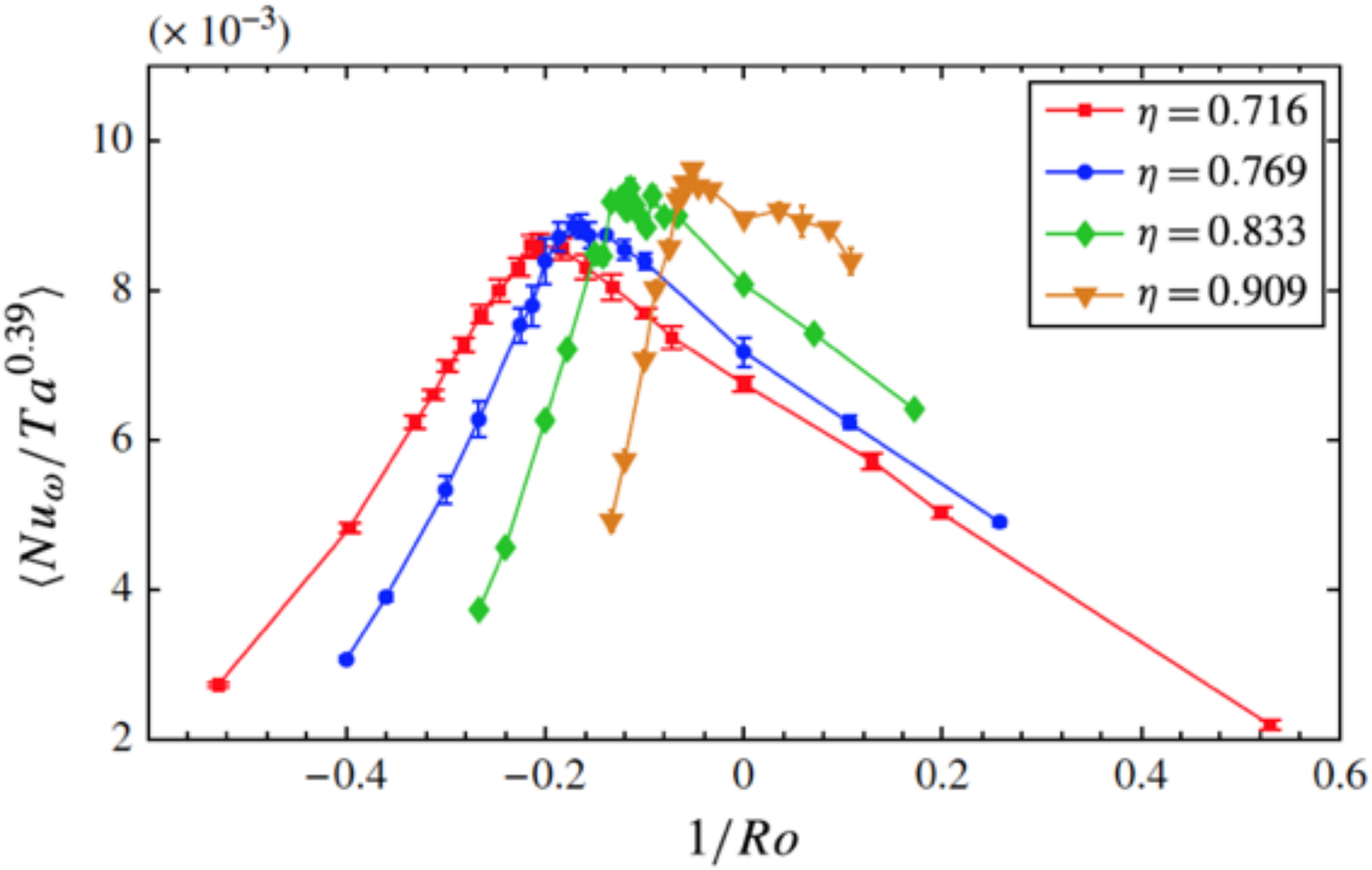




Bulk gradient can be controlled also through η!

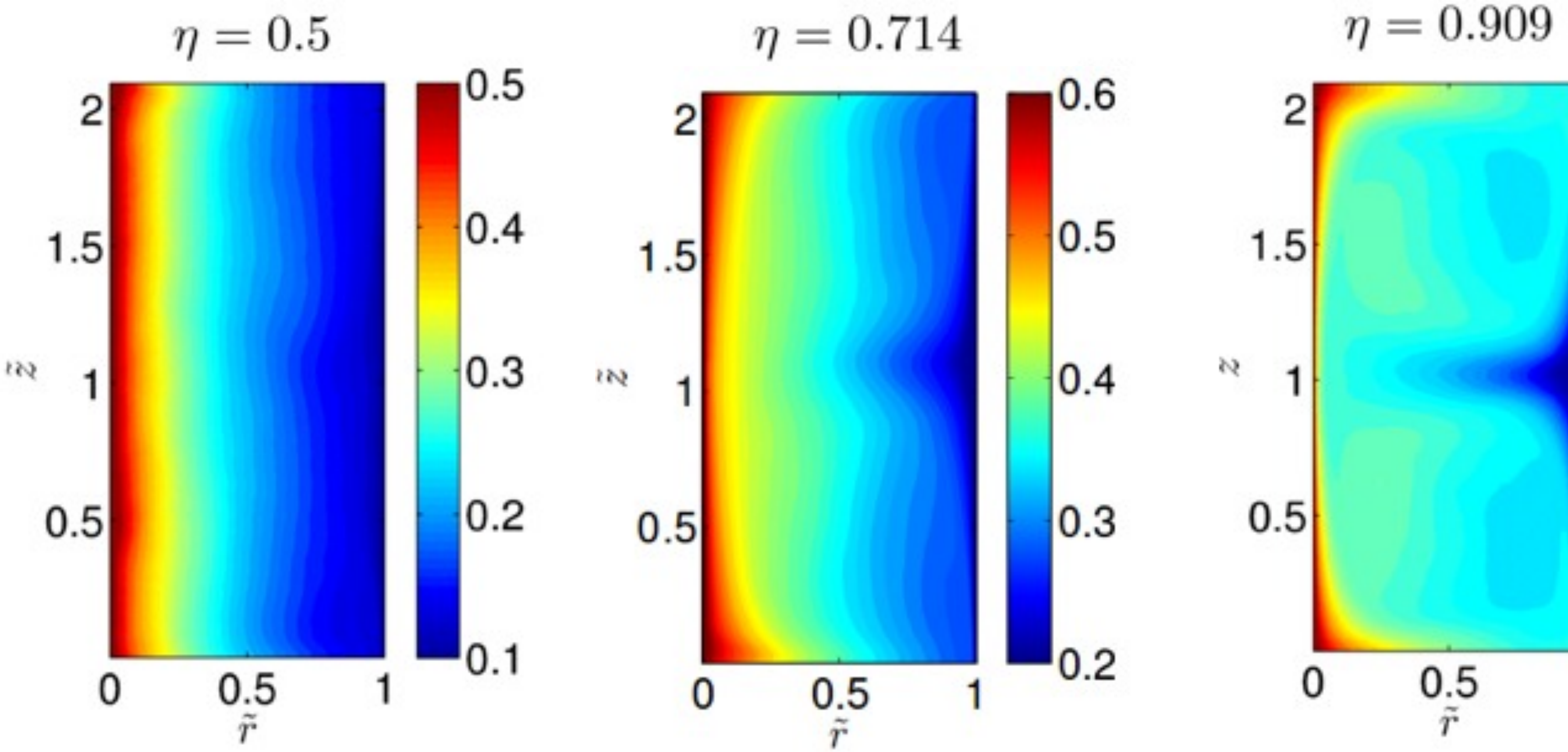
Large η has the best transport properties in ultimate regime

Transport from large Ta experiments at different η



Large η has the best transport properties in ultimate regime

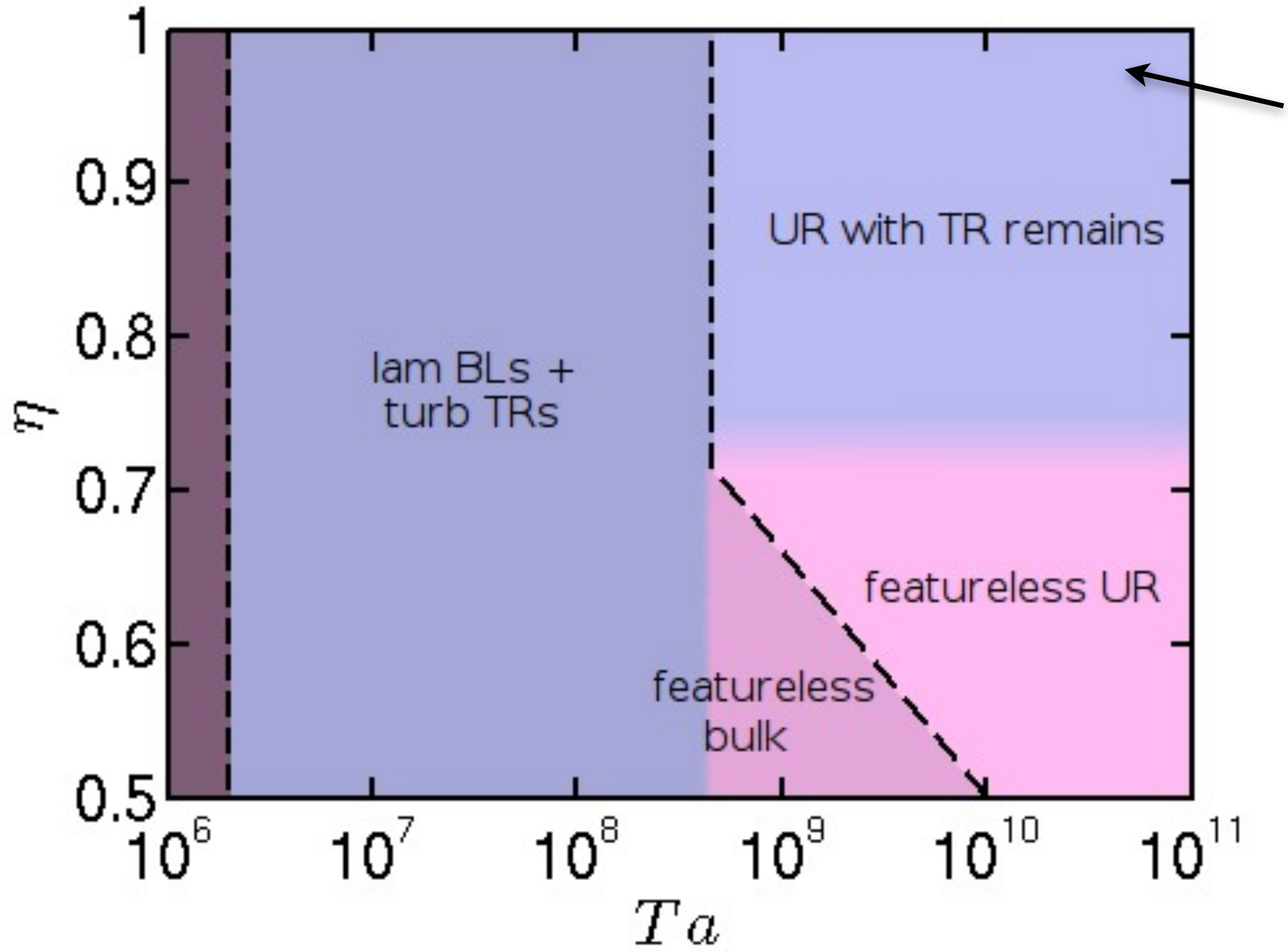
Increasing η -> more pronounced roll structure -> better transport



Just as decreasing 1/Ro in the weakly counter-rotating regime

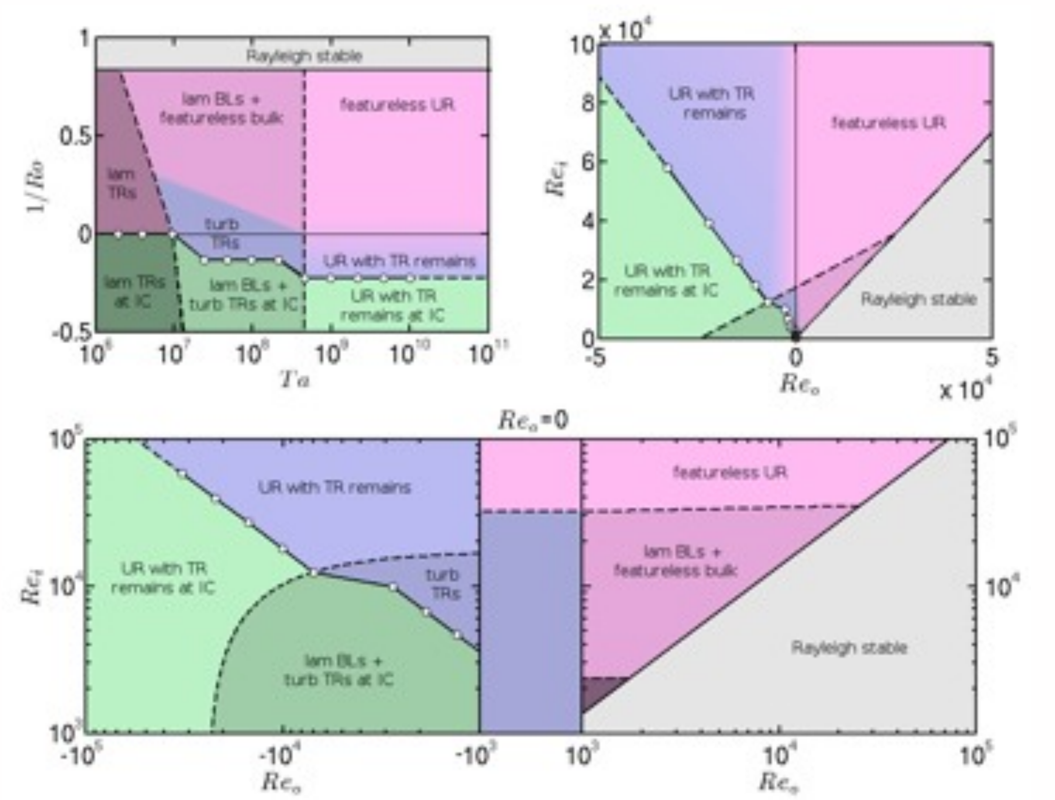
$Ta = 10^{10}$ $Ro^{-1} = 0$

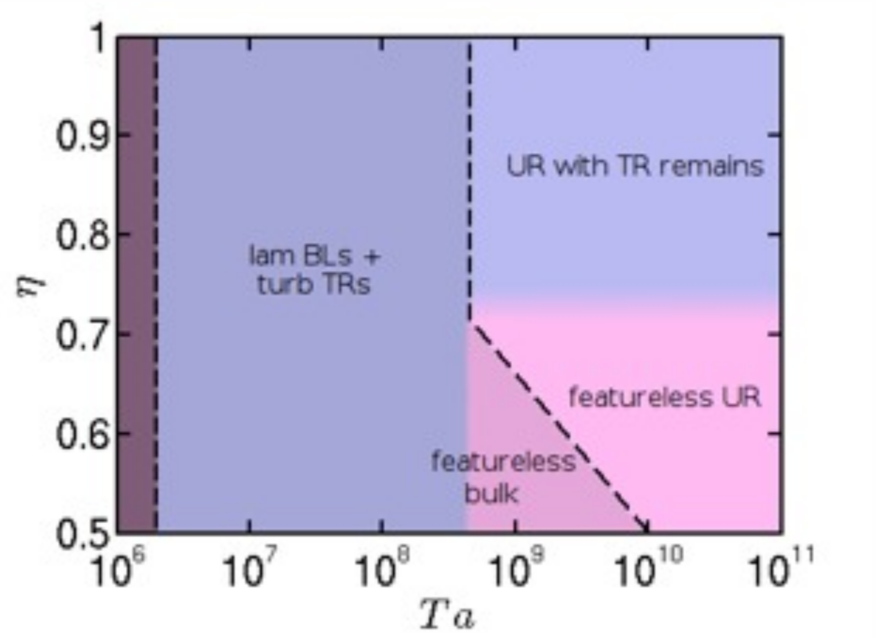
Phase diagram of η-dependence



$Ro^{-1} = 0$

Preliminary conclusions





(here $Ro^{-1} = 0$)

(here η = 0.714)

Startup behavior (inner cylinder) in numerics and experiments

(outer cylinder at rest)

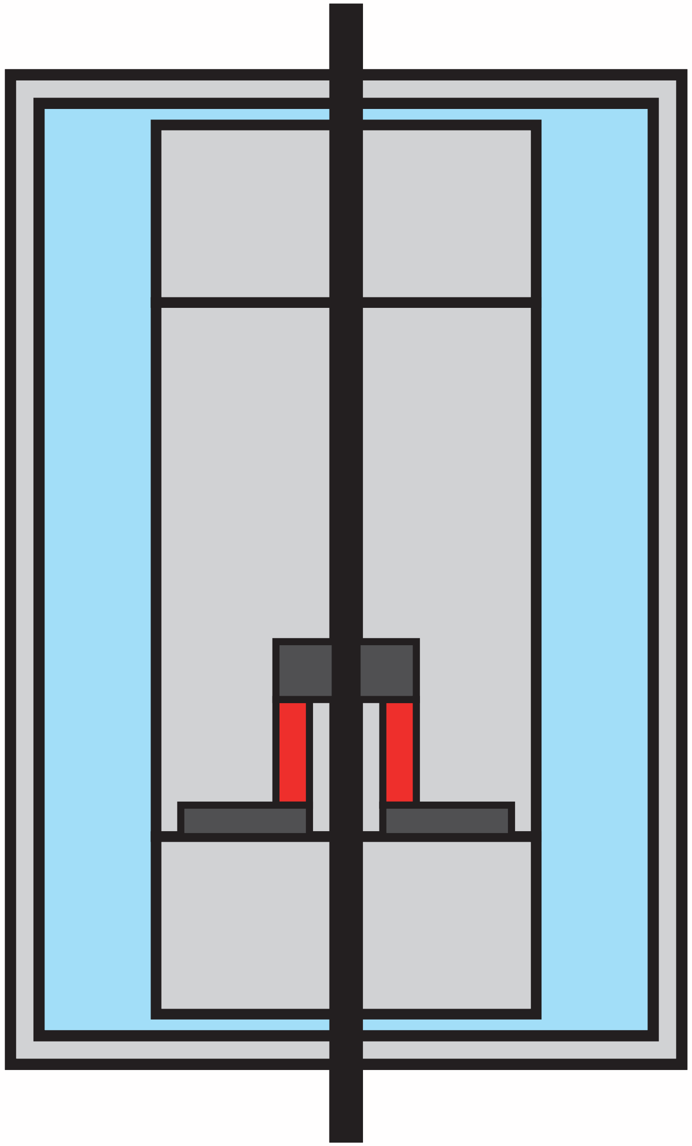
$\eta = 0.714$ $Ro^{-1} = 0$

$Ta = 10^8$

But this is not the full story yet...

Optimal transport and turbulent structures

Torque sensor

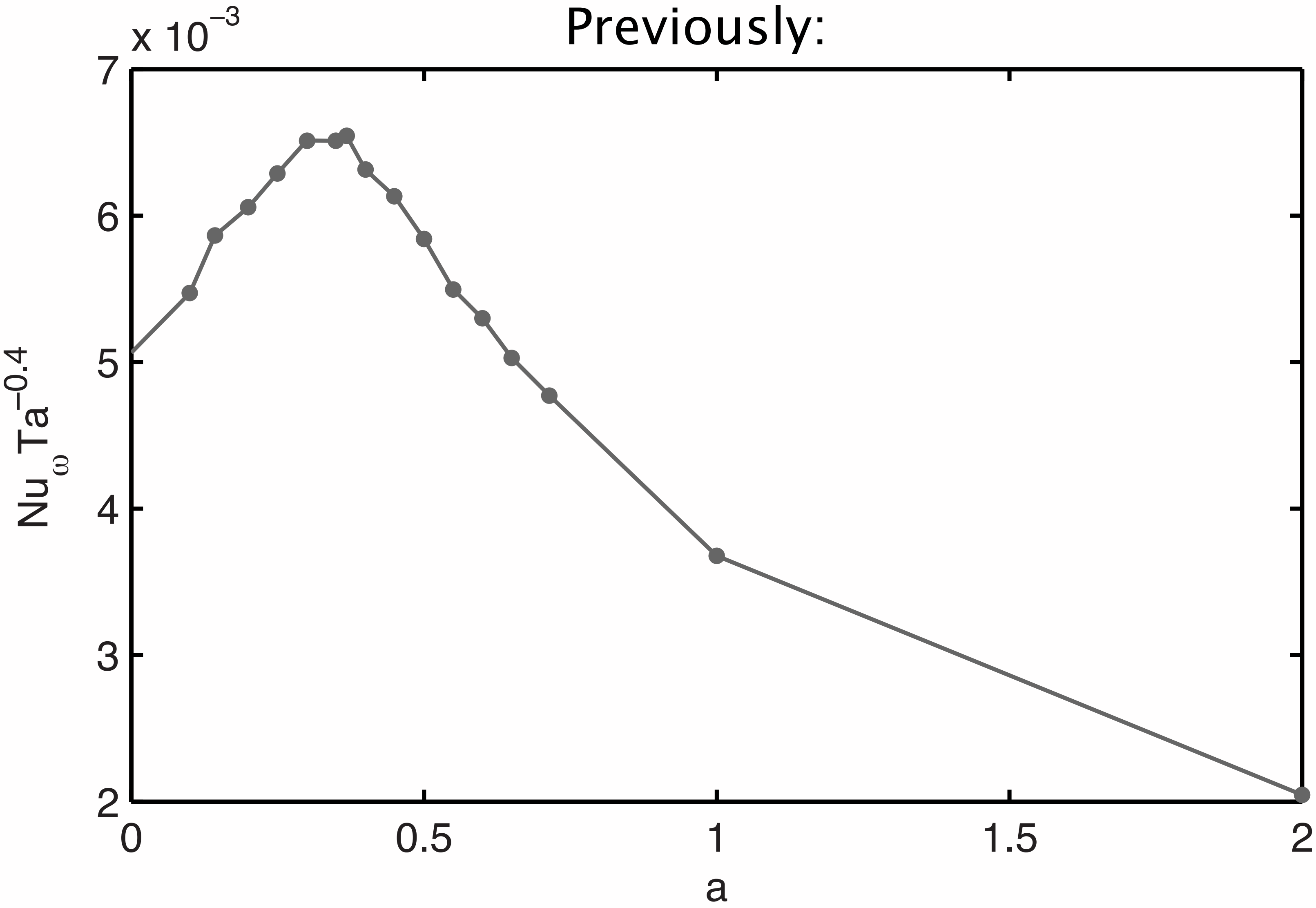


Previously:

A co-axial torque transducer

Designed to measure torque

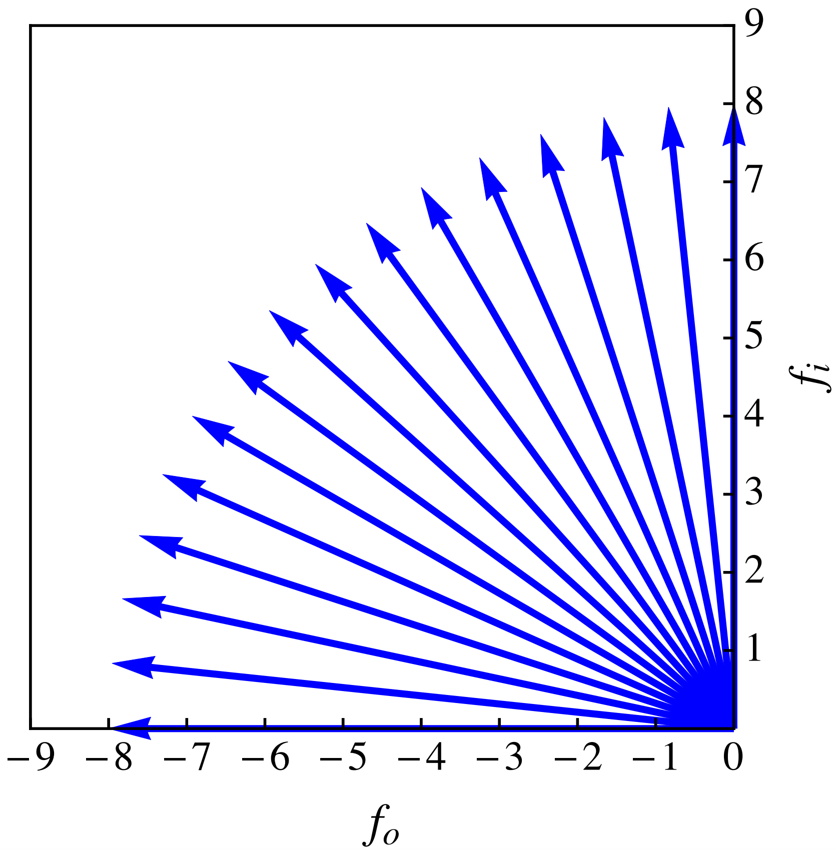
Optimal transport



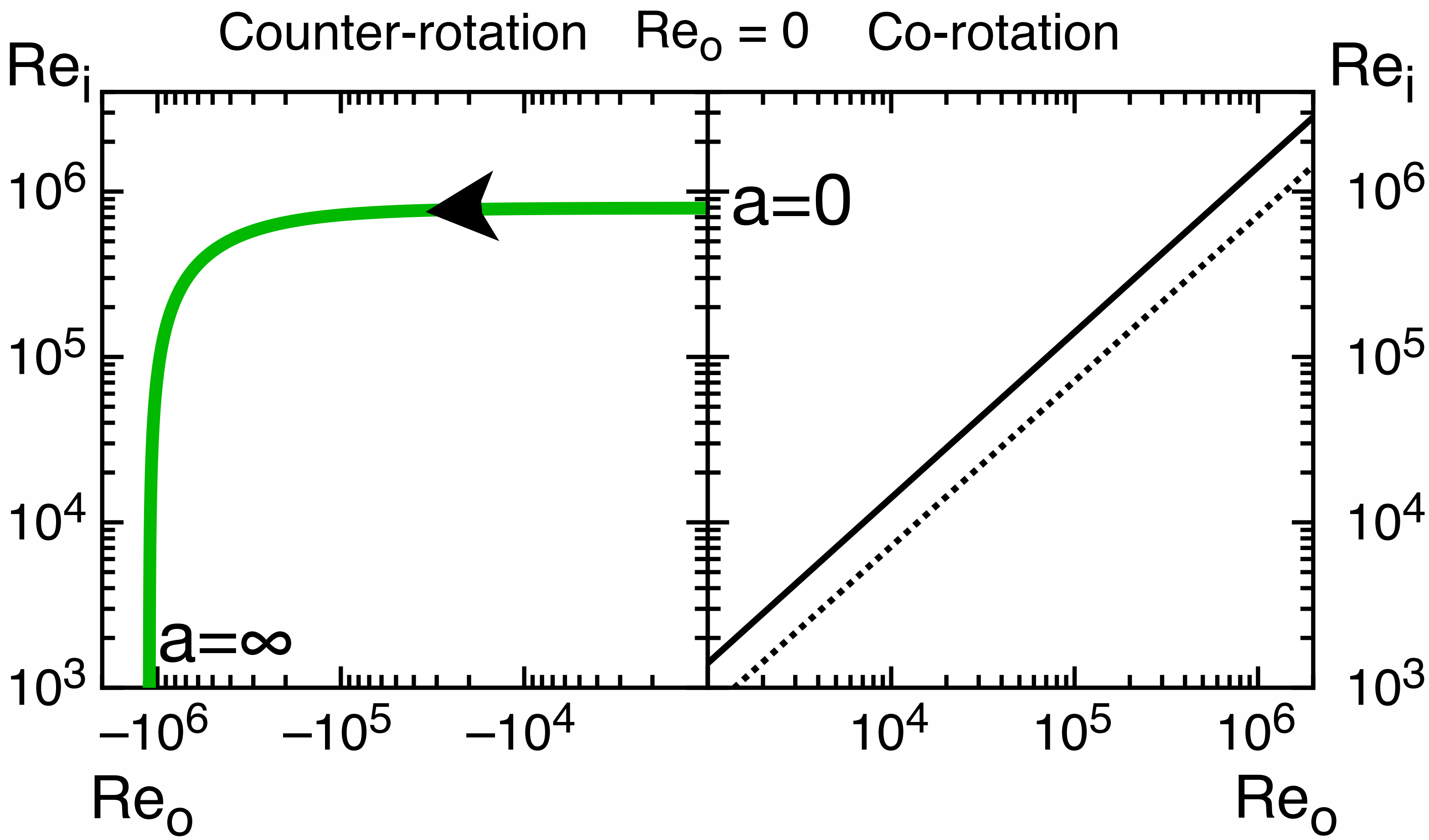
Measurement procedure

Previously:

for every a we vary Ta and measure Nu_ω
measure 3 hour per 'a'



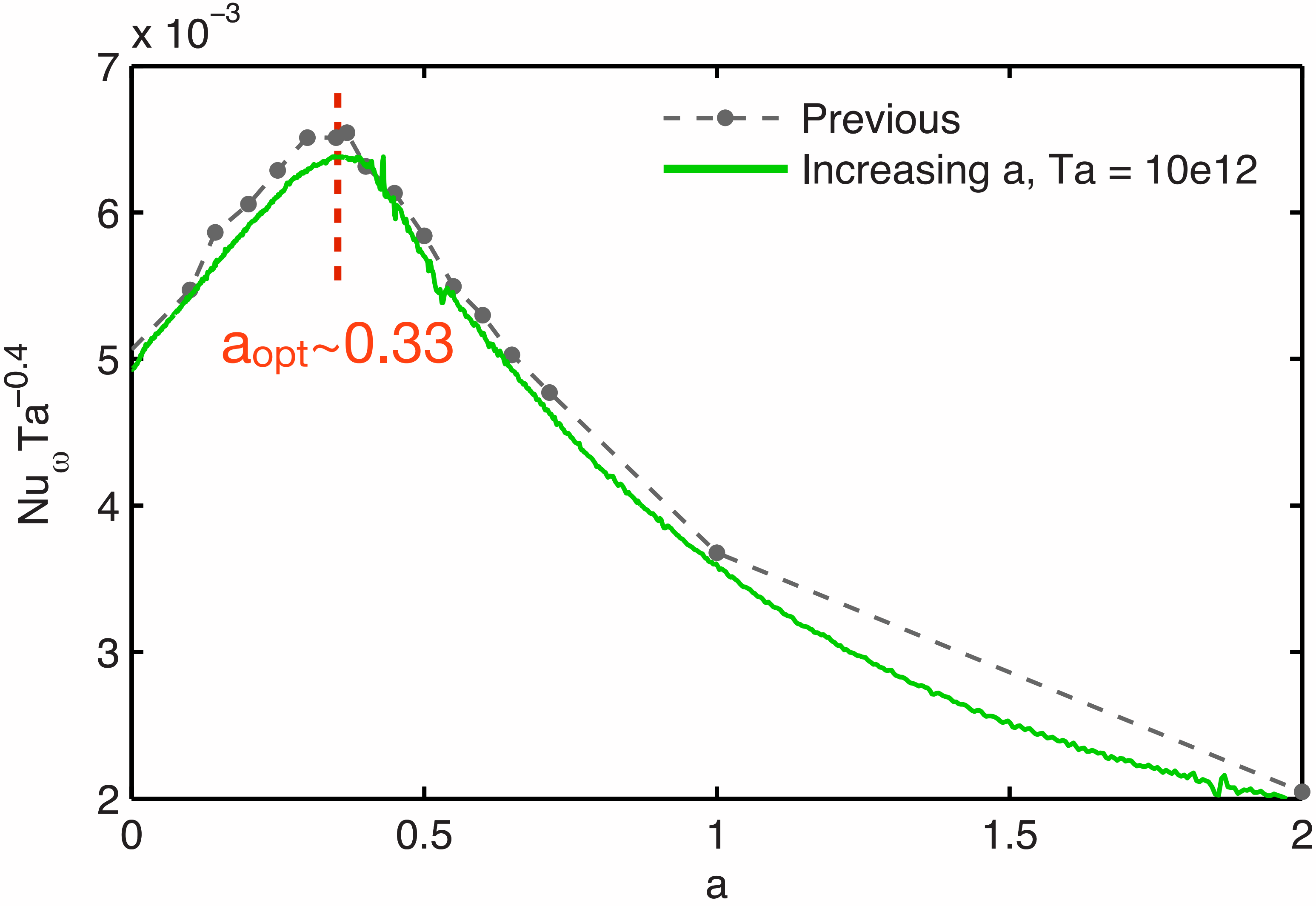
Measuring Nu with increasing a



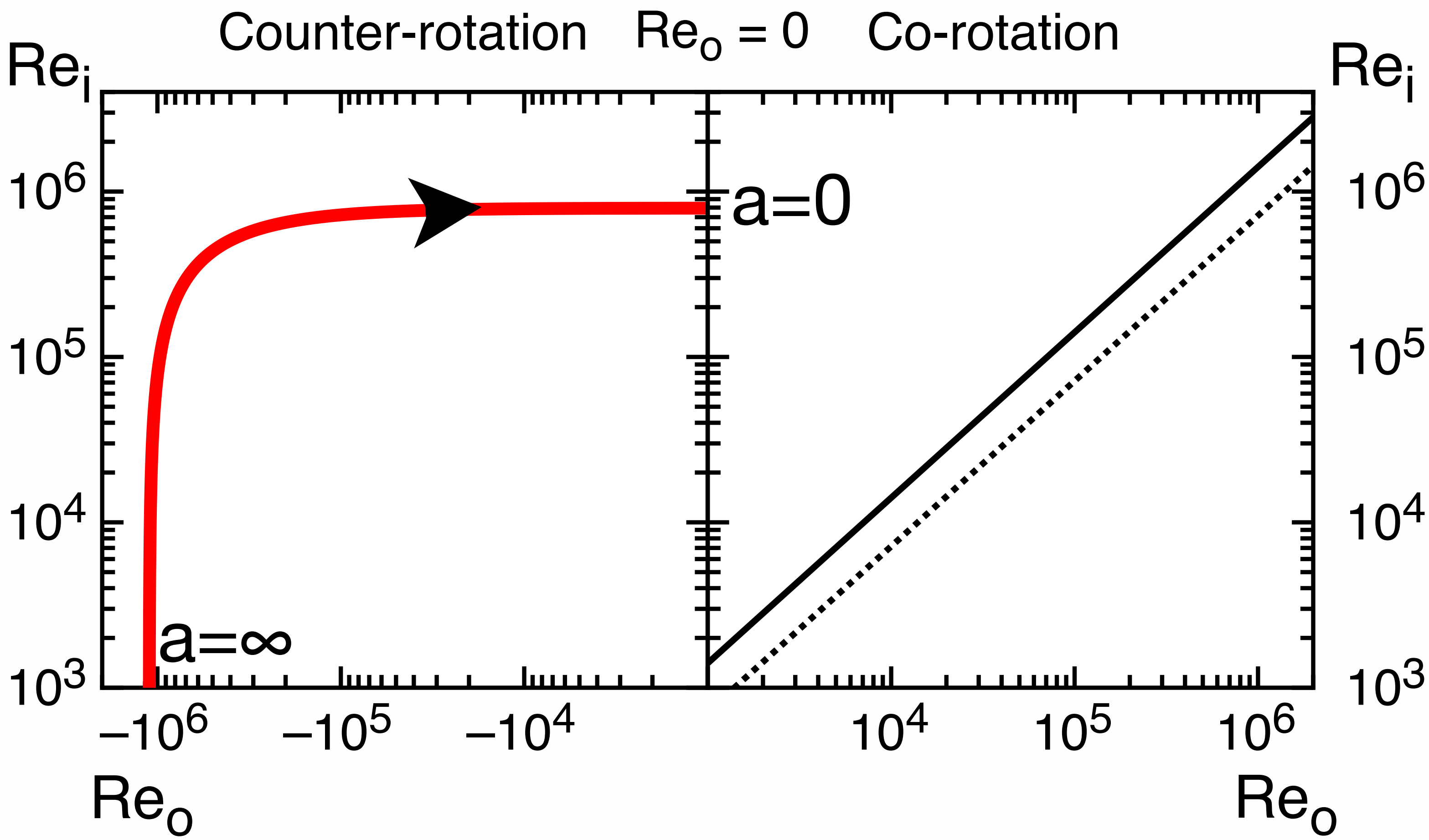
Fixed Ta number !

$$Ta = \frac{1}{4}\sigma(r_o - r_i)^2(r_i + r_o)^2(\omega_i - \omega_o)^2\nu^{-2}$$

Measuring Nu with increasing a

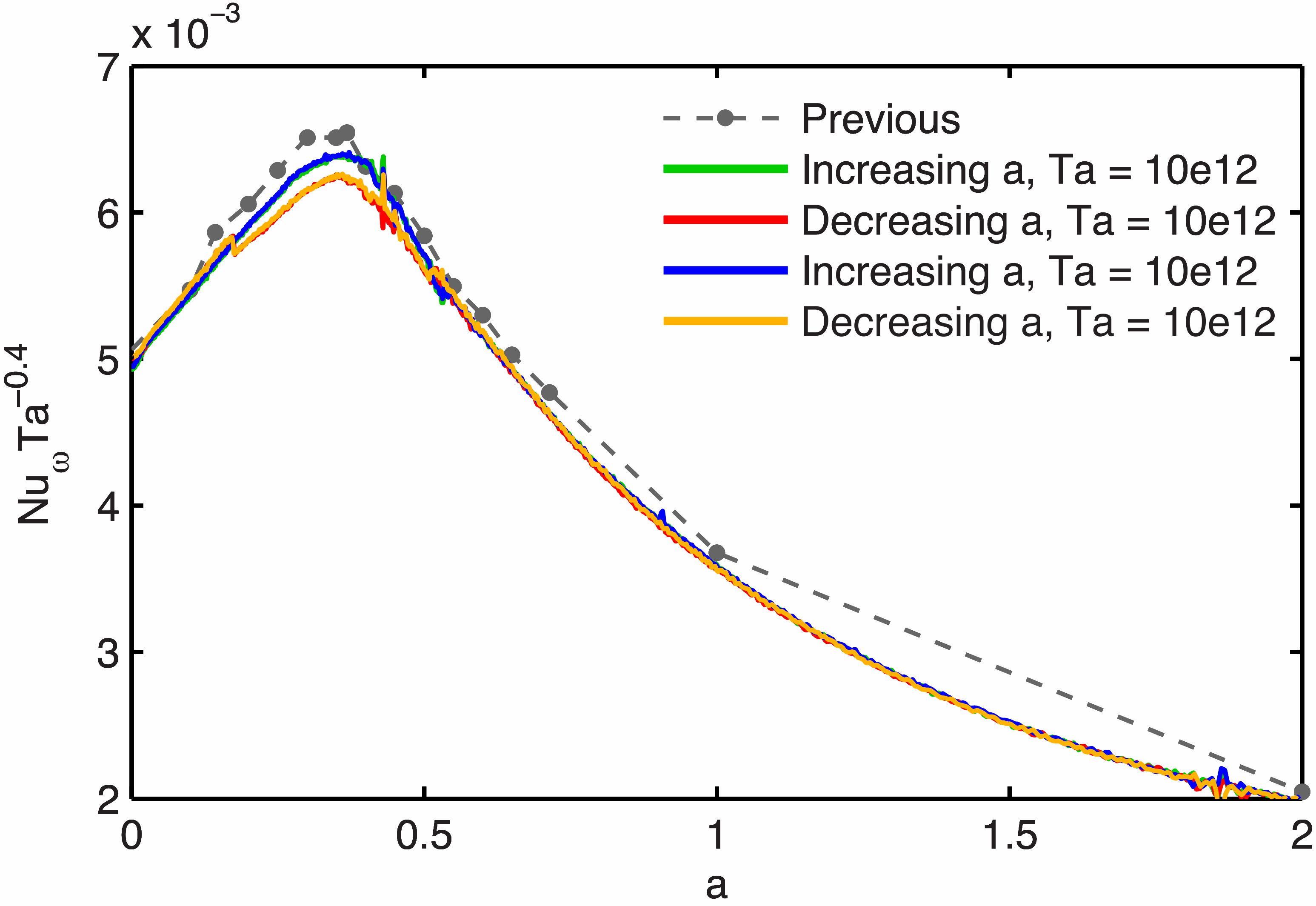


Measuring Nu with decreasing a

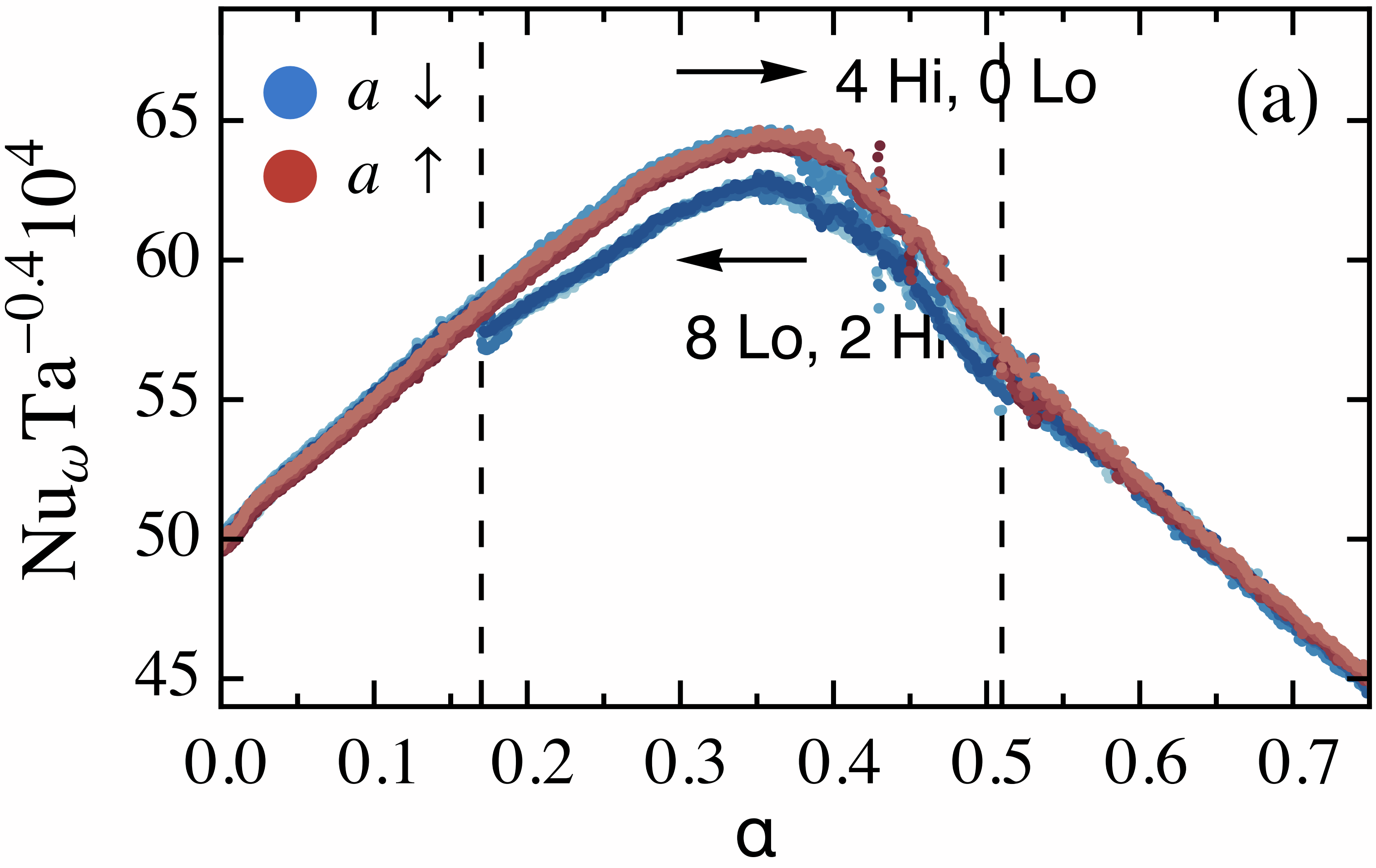


Fixed Ta number !

Measuring Nu with increasing/decreasing a



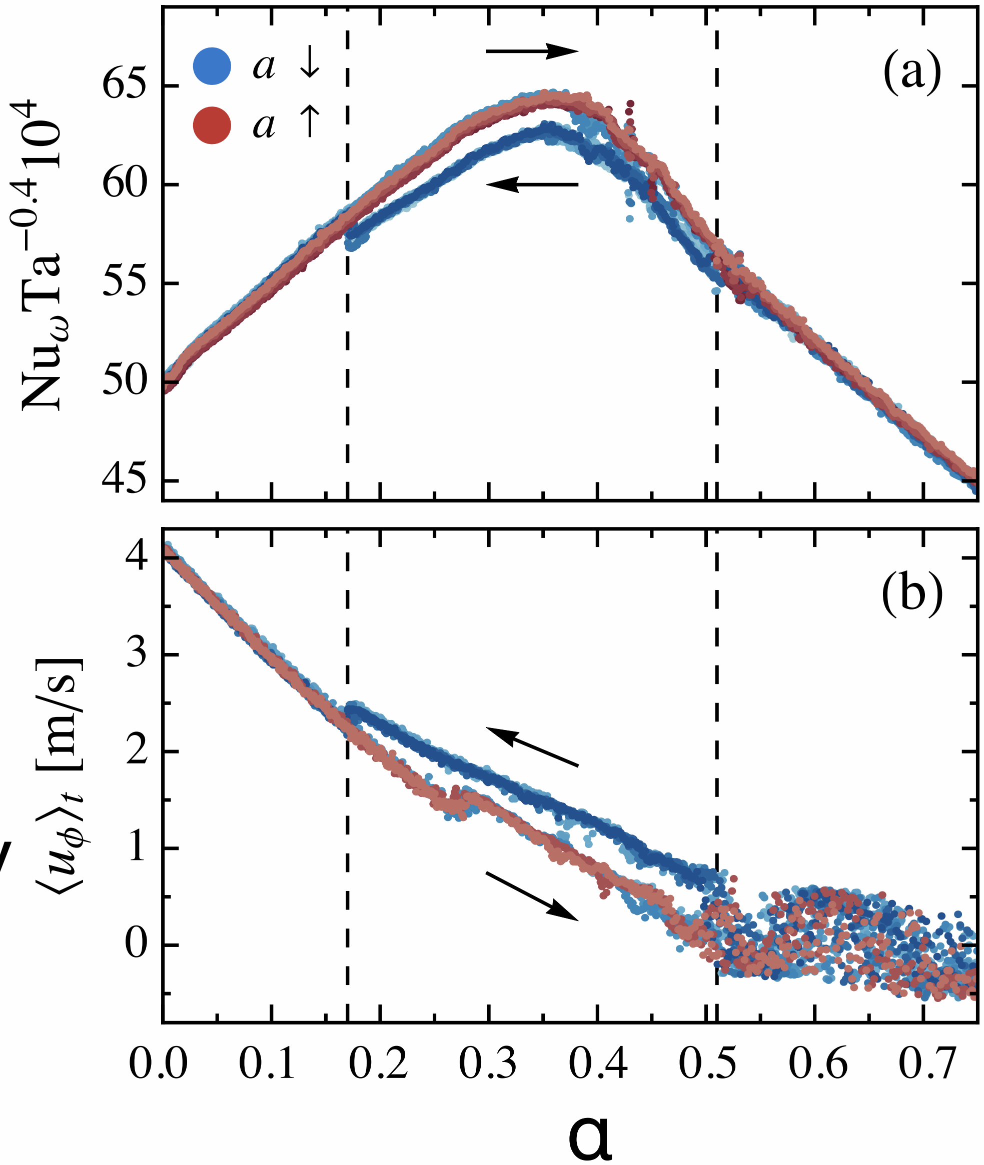
Multiple turbulent states appear around the optimal a



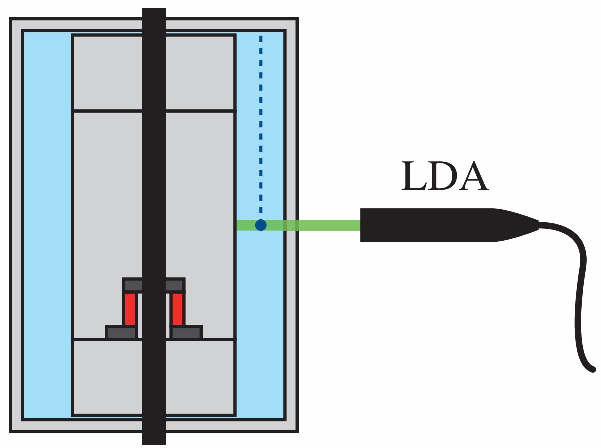
Global transport and internal flow

Global torque

Local flow velocity



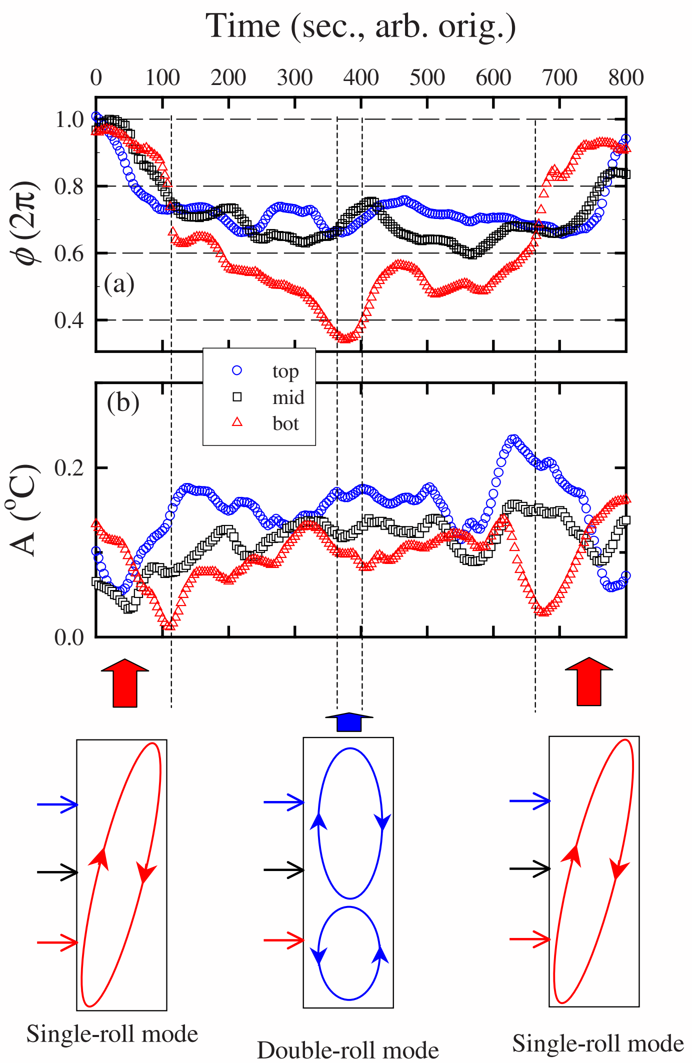
Local flow velocity also shows two states !



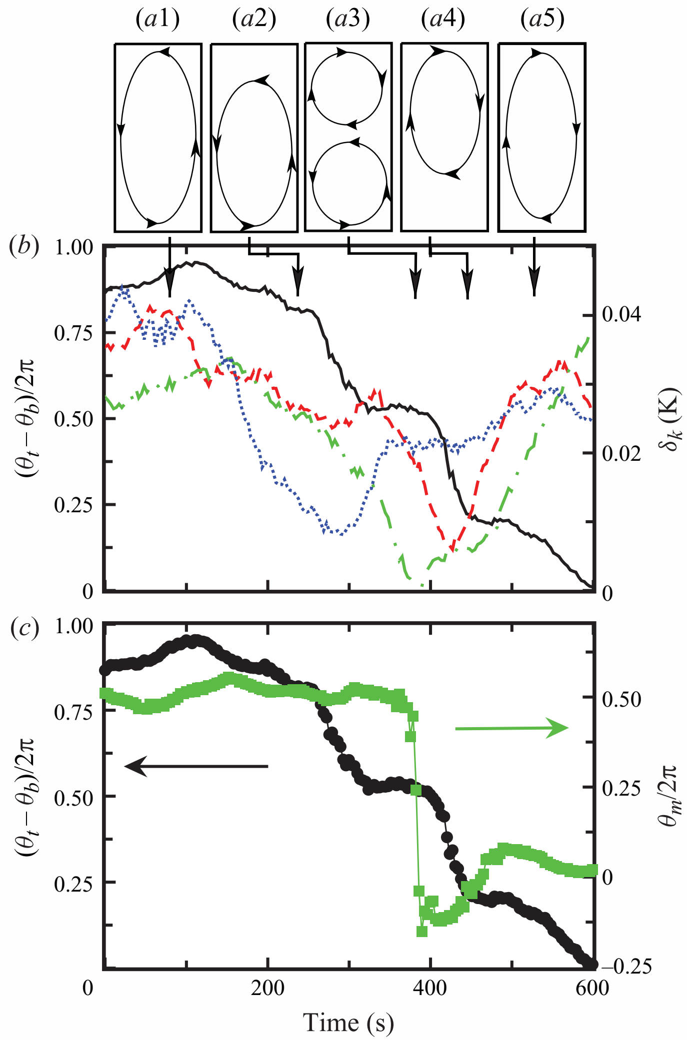
Flow structures?

Multiple turbulent states !!!!

RB flow in classical turbulent state



Xi & Xia, Phys. Fluids, 20, 055104 (2008)

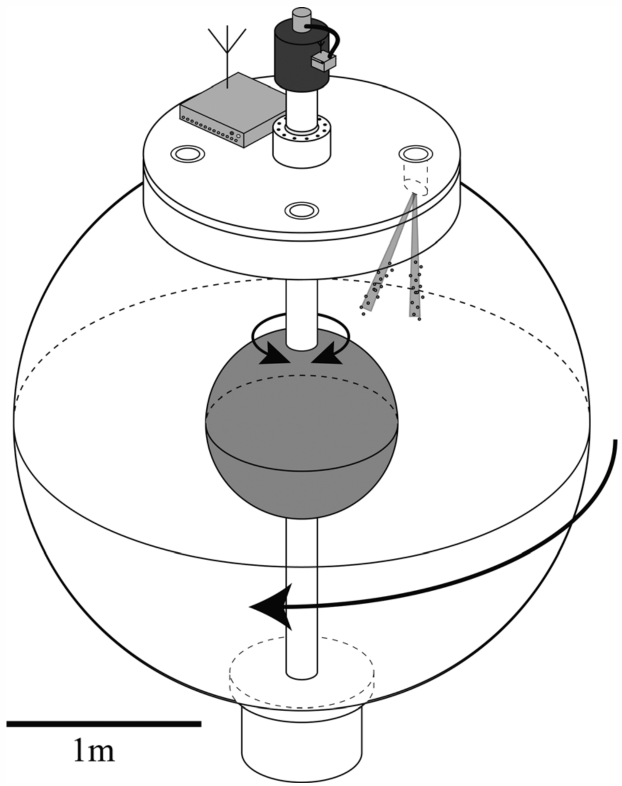


Weiss & Ahlers, J. Fluid Mech. 676, 5 (2011)

Multiple states:

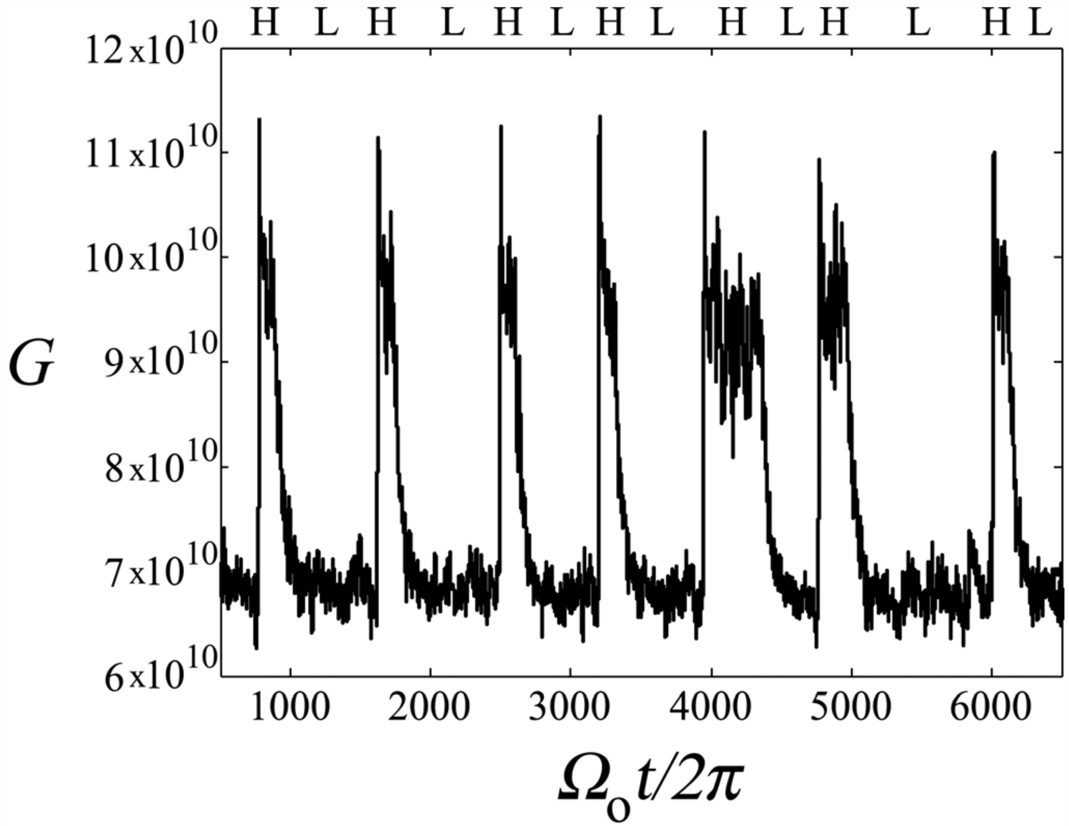
continuous switching between two different roll states with different heat transfer properties

Rotating spherical Couette flow

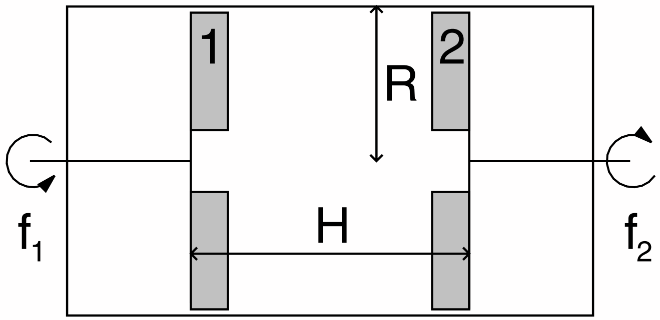


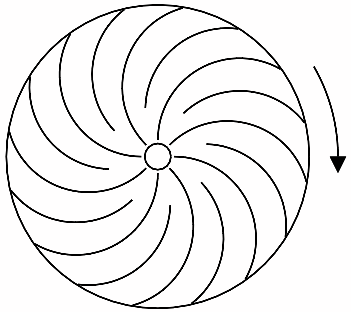
Zimmermann, Triana, Lathrop,
Phys. Fluids, 23, 065104 (2011)

Spontaneous switching between two turbulent states

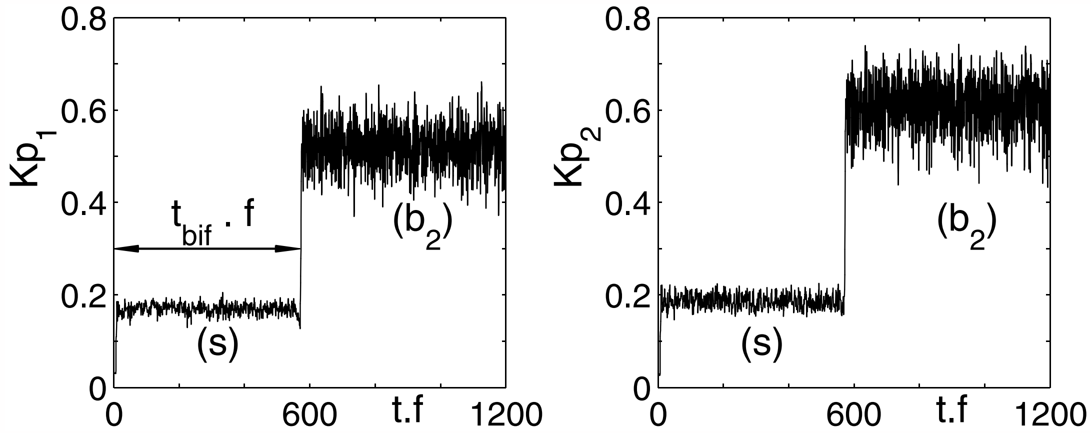


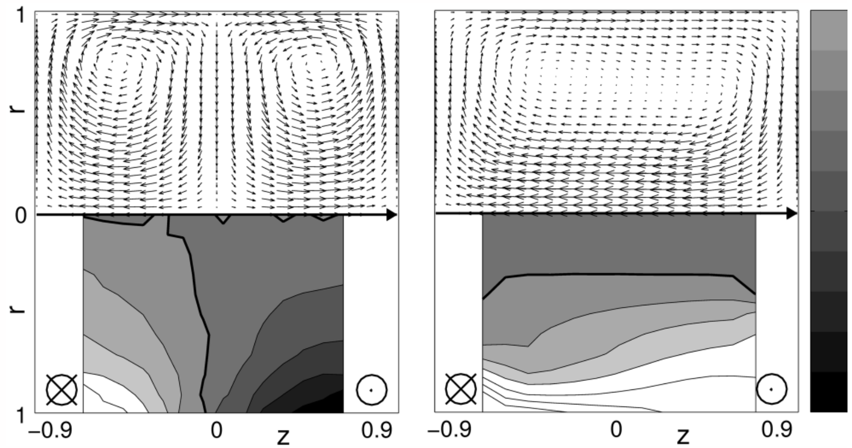
von Kármán flow with curved blades





Global Bifurcation in a highly turbulent von Kármán flow



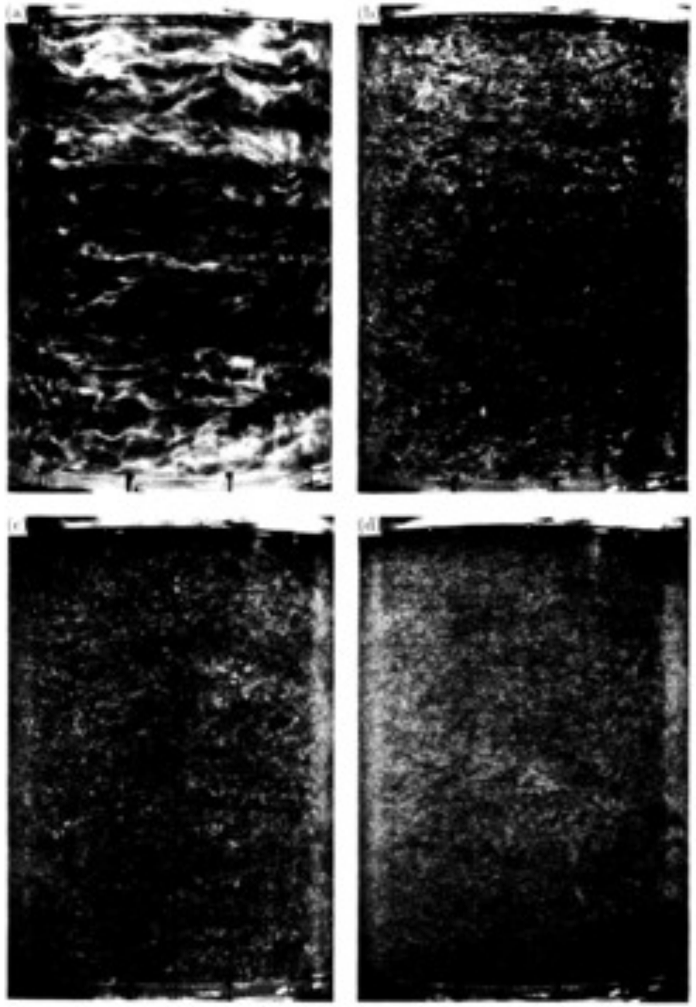


Ravelet, Marié, Chiffaudel, Daviaud, Phys. Rev. Lett., 93, 164501 (2004)

Do stable turbulent structures exist in ultimate TC flow?

Structures in high-Reynolds-number TC flow

Re=6000 Re=24000



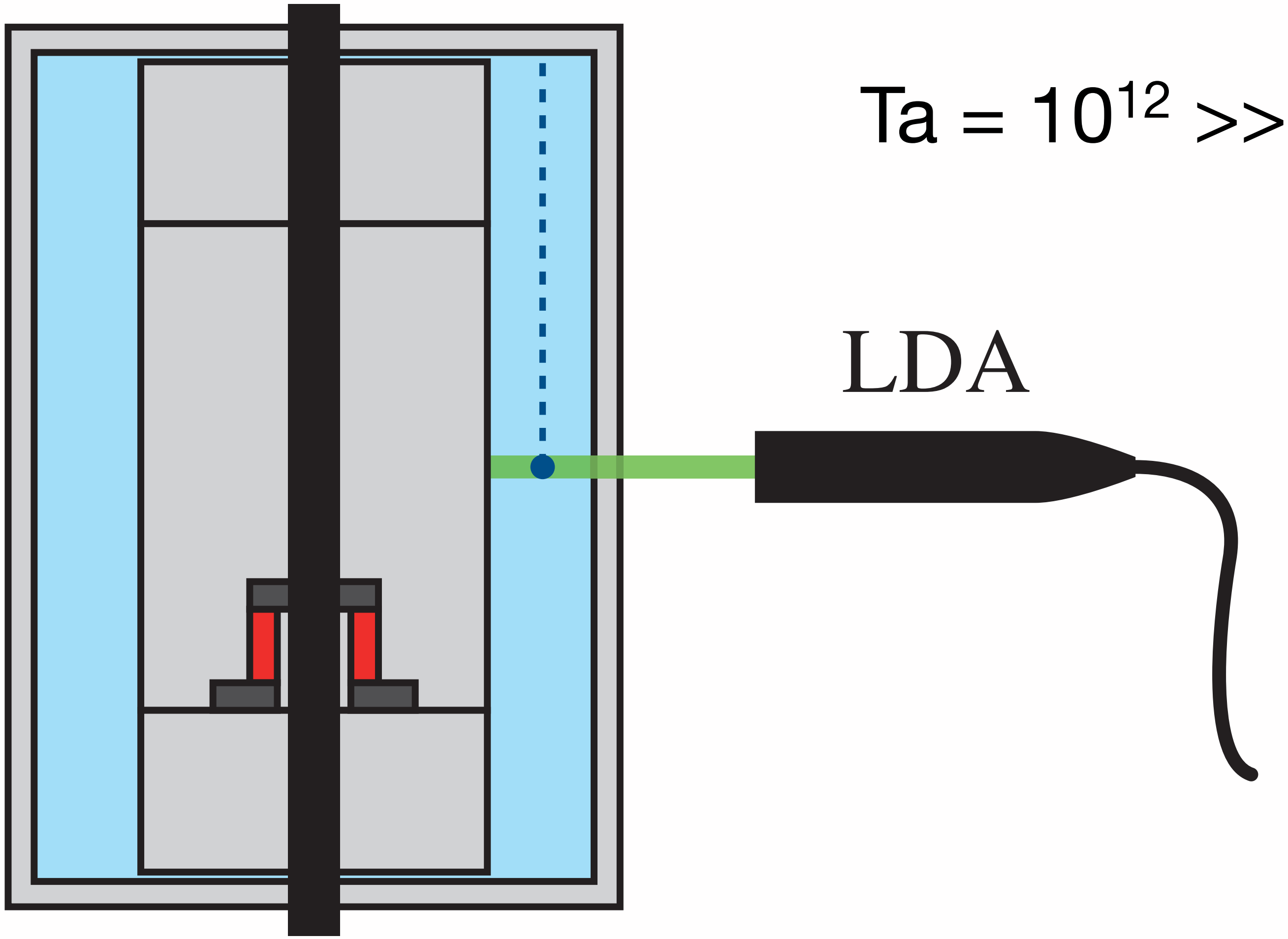
Re=48000 Re=122000

Only inner cylinder rotation: a=0

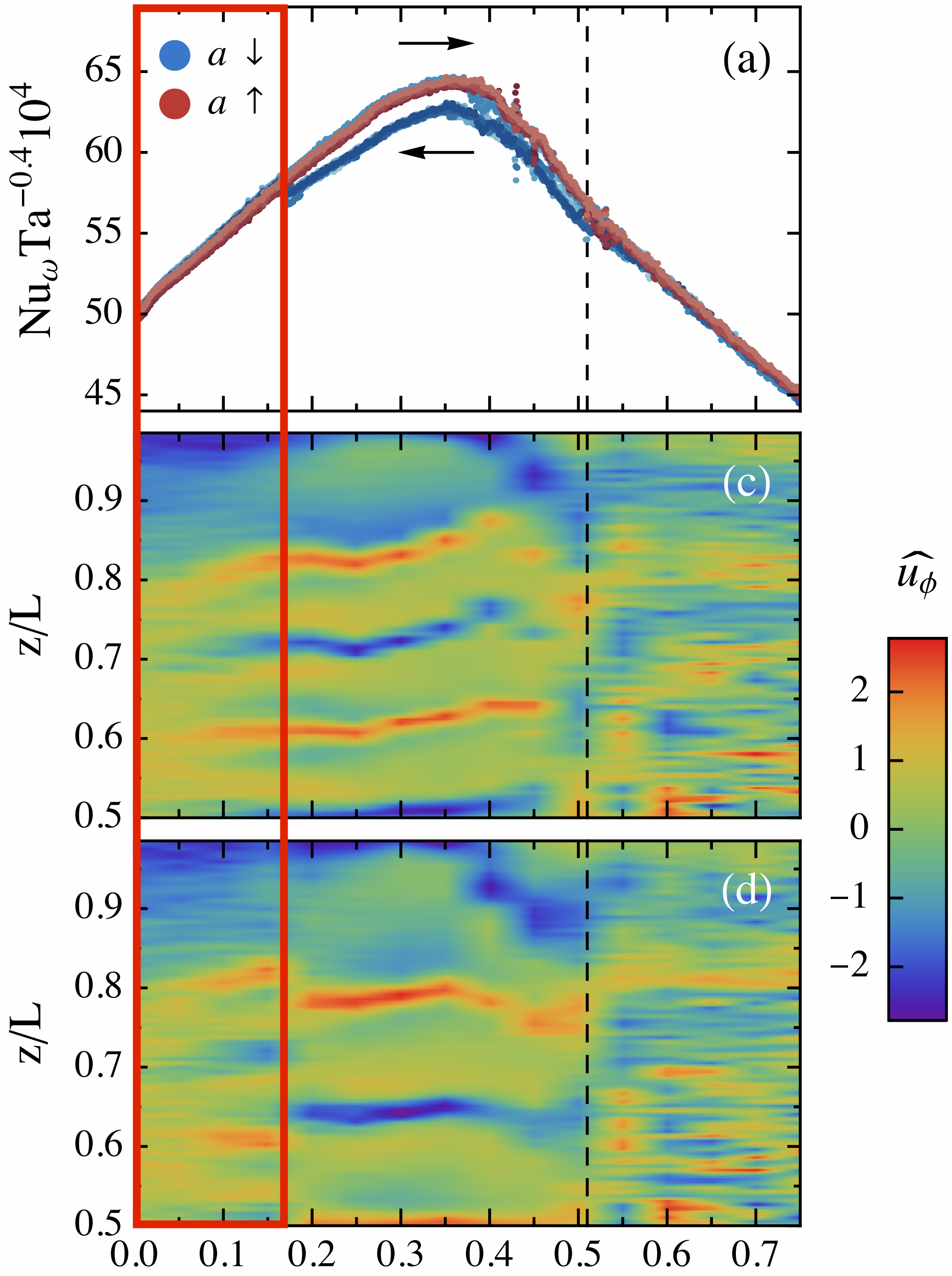
Stable turbulent vortices vanish at Re > 10^5

Lathrop, Fineberg & Swinney, Phys. Rev. A 46, 6390 (1992)

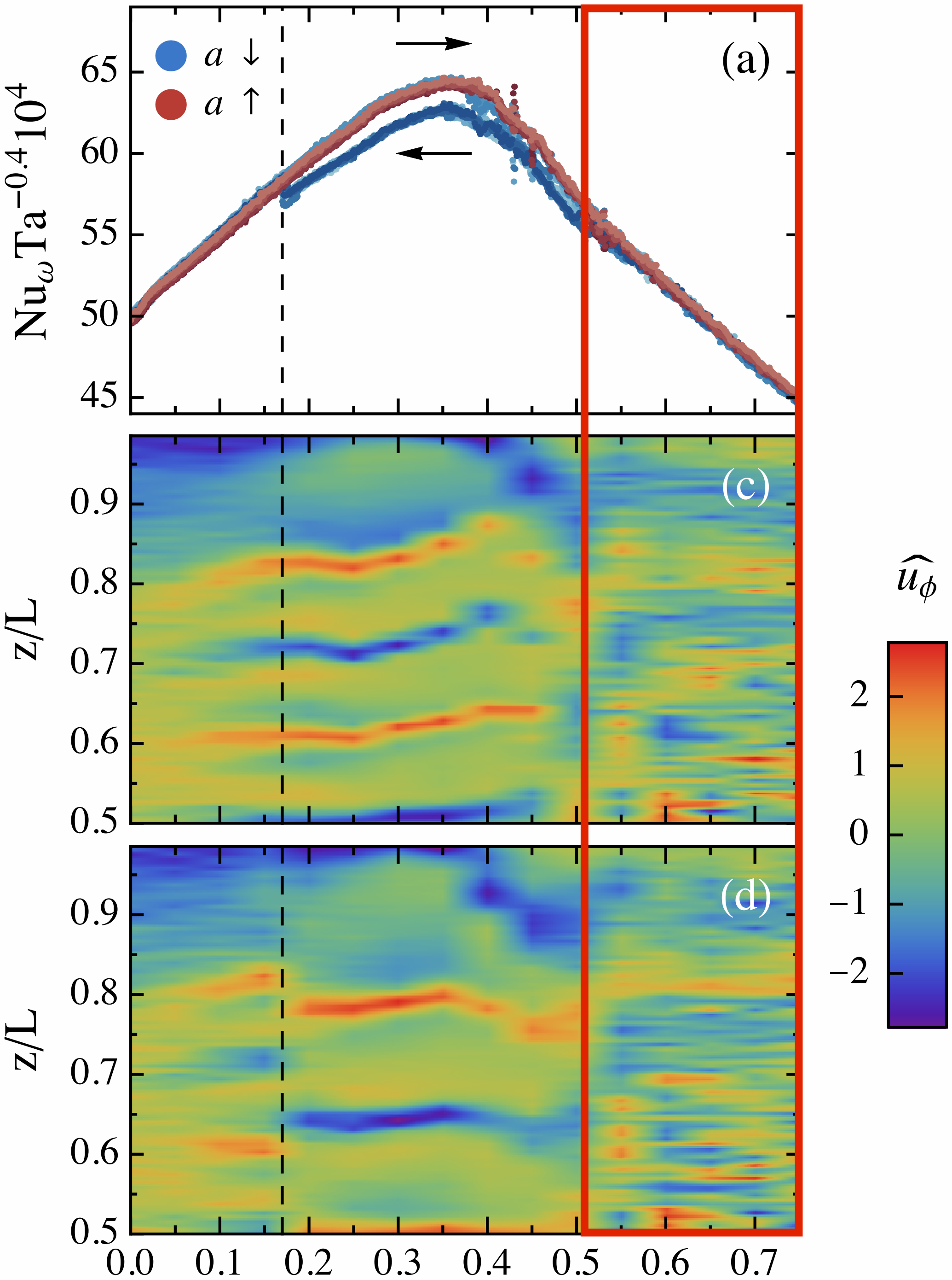
Measure axial dependence



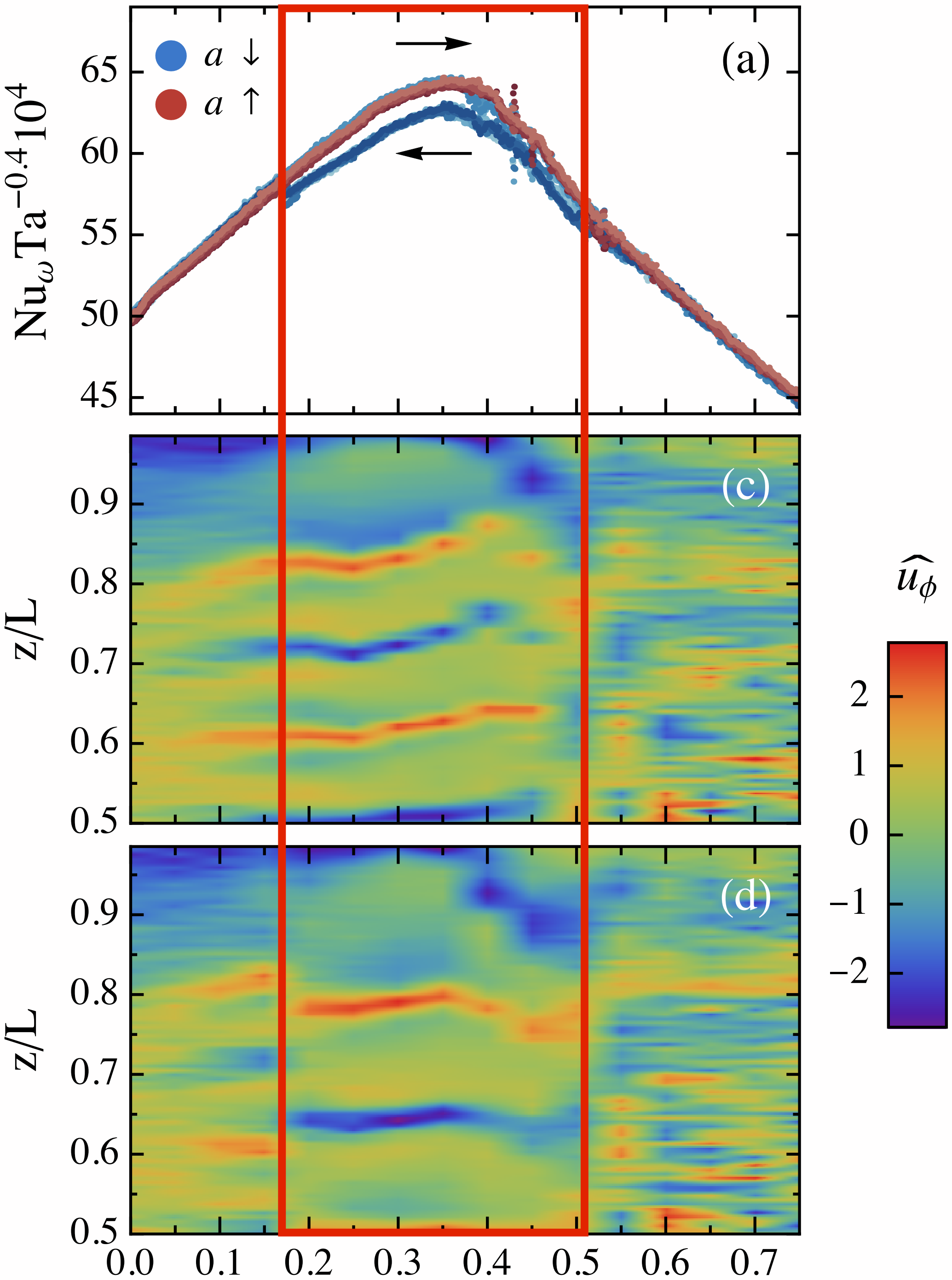
$Ta = 10^{12} >> Ta_c$



No strong turbulent structures for 0<a<0.17

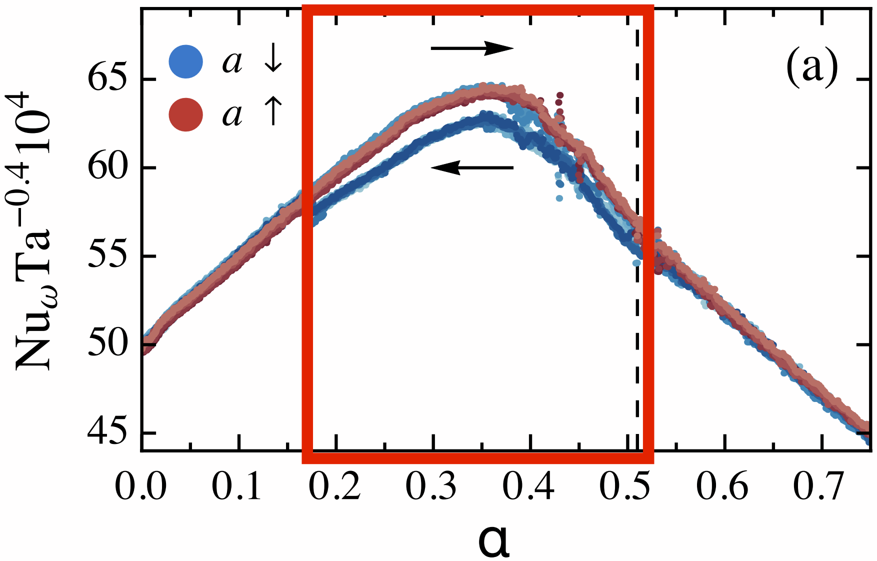


No stable turbulent structures: a>0.51

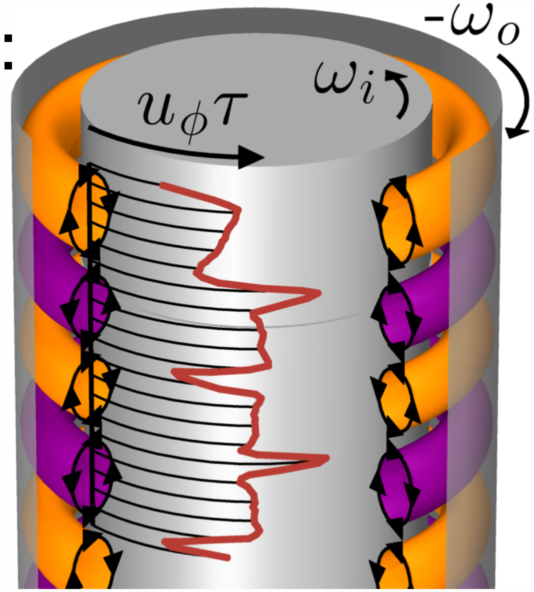


Multiple state regime: 0.17<a<0.51

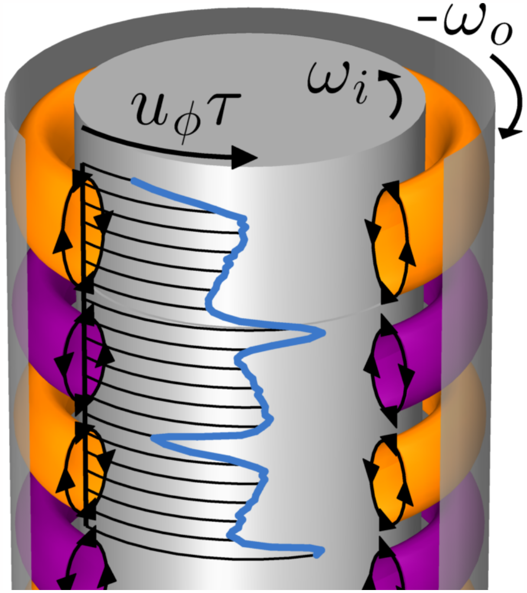
Stable turbulent structures are observed!



High state:
8 rolls



Low state:
6 rolls

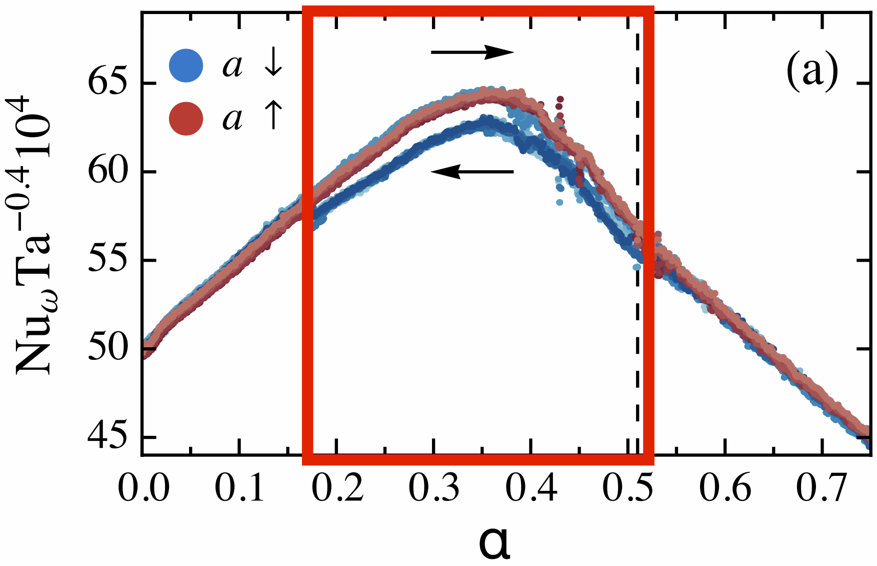


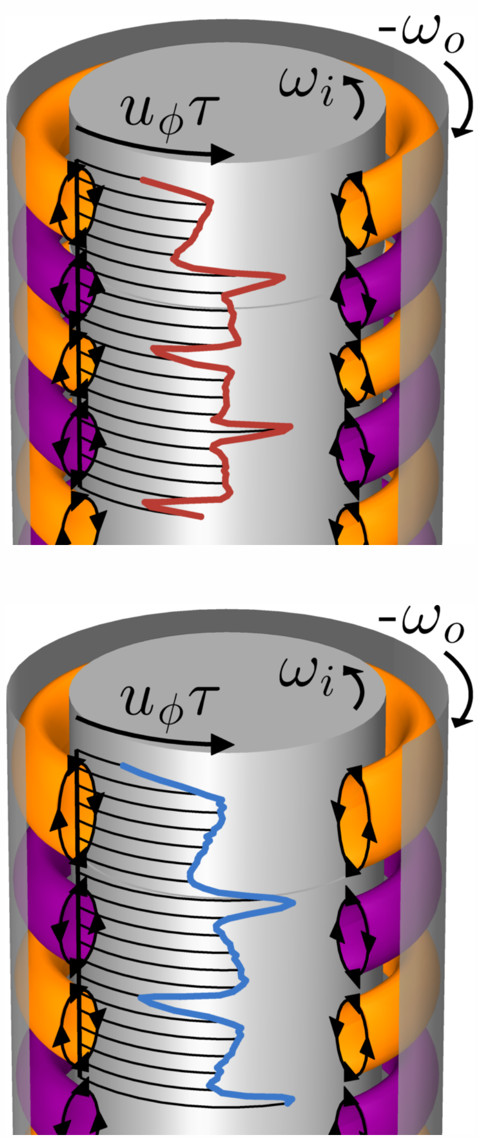
Global and local measurements: multiple turbulent states exist even at $Re=10^6$ ($Ta=10^{12}$)

Summary of 2nd part

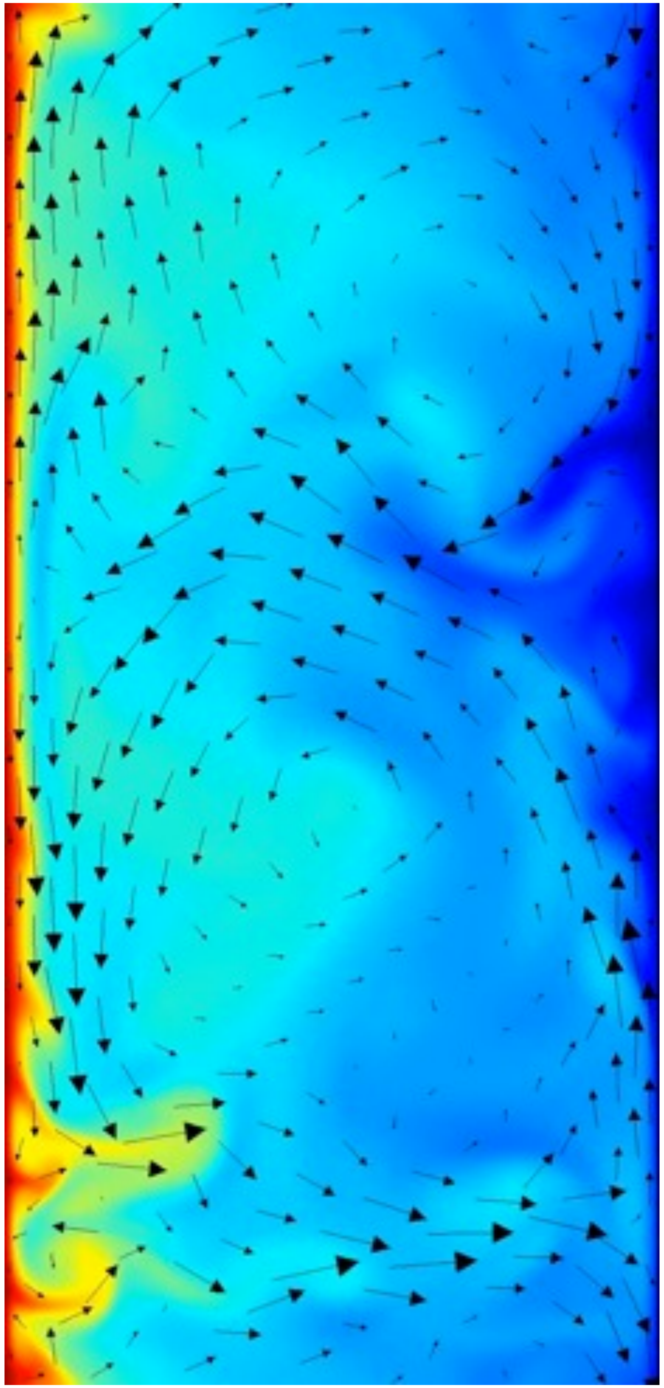
Multiple turbulent states exist even at Re=10^6 (Ta=10^{12})

Optimal transport is connected to the existence of the large-scale coherent structures





Huisman, van der Veen, Sun & Lohse, Nat. Commu. 5, 3820 (2014)



Start with a turbulent flow and add Coriolis force

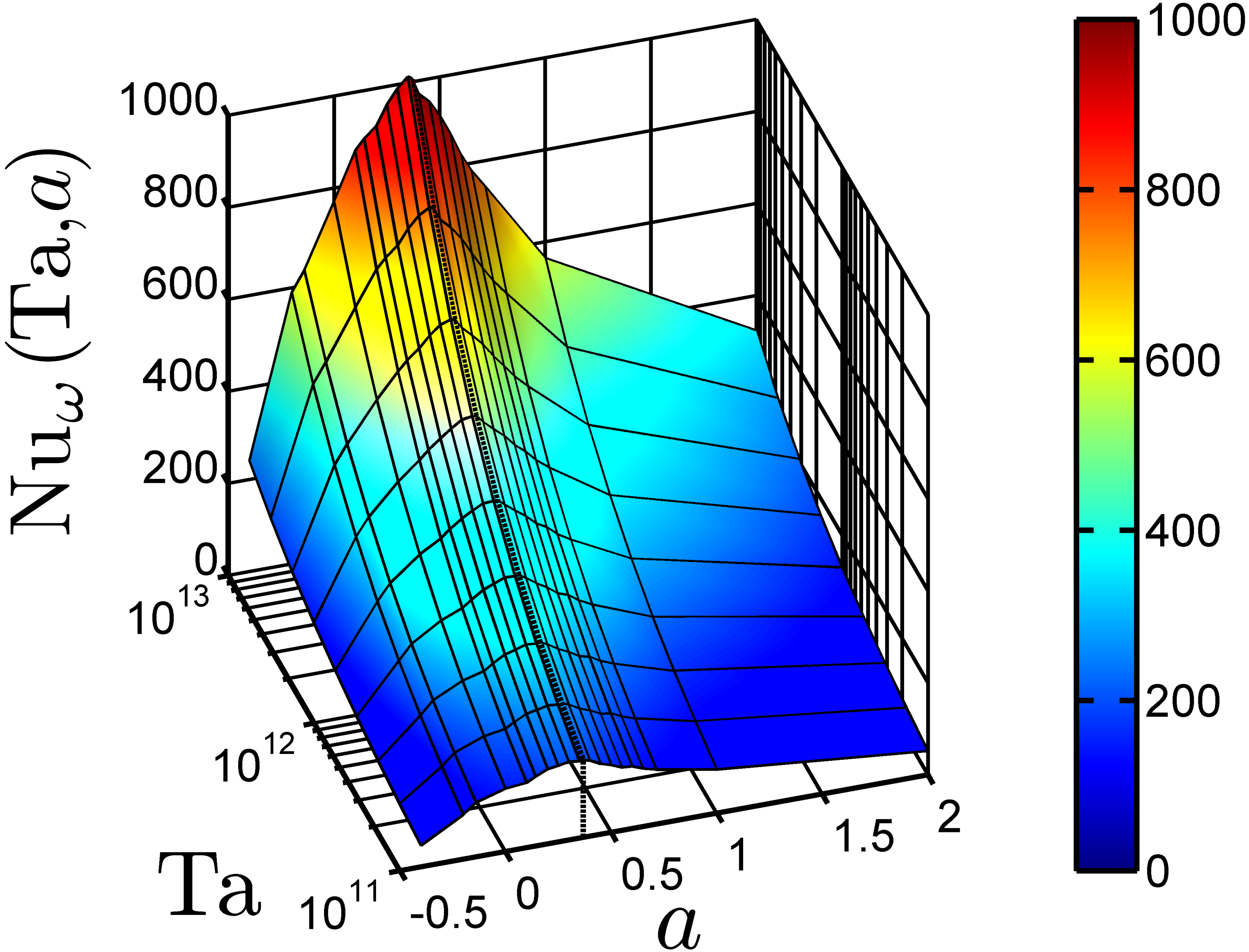
$$\eta = \frac{r_i}{r_o} = 0.714$$

$$\Gamma = \frac{L}{r_o - r_i} = 2.03$$

$$Re_s = \frac{r_i(\omega_i - \omega_o)d}{\nu} = 8020$$

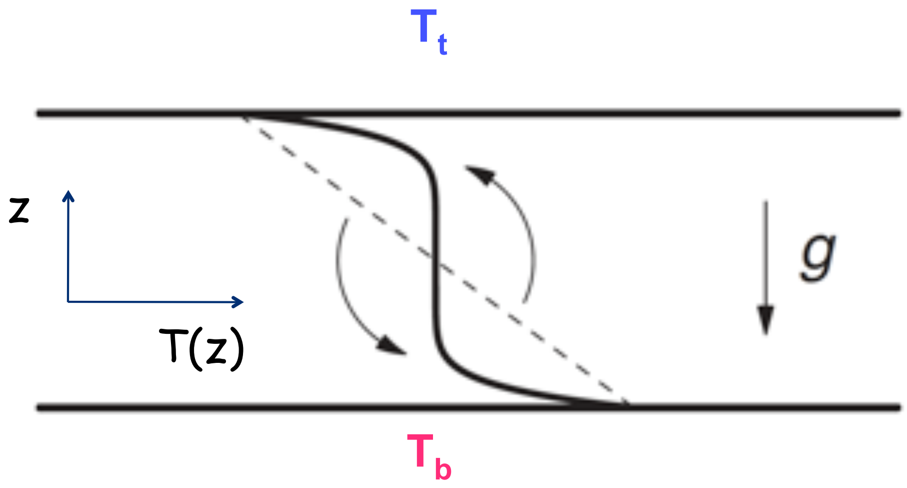
$$Ro^{-1} = 1.22$$ (quasi-Keplerian)

Global measurements: Nu(Ta,a)

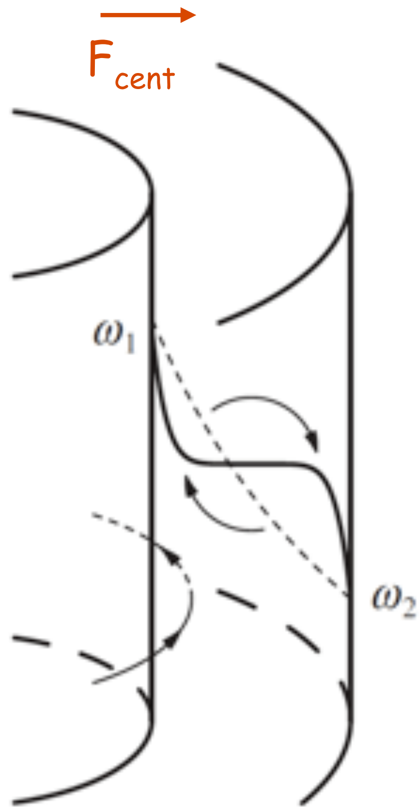


Compare RB & TC flow

Analogy RB and TC



Wind determines T(z) profile:
• Thermal BL width λ
• Kinetic BL witdth δ



Wind determines ω(r) profile:
• Vorticity BL width λ
• Kinetic BL witdth δ

Eckhardt, Grossmann, Lohse,
J. Fluid Mech. 575, 221 (2007)

RB

Conserved: heat flux

$$J = \langle u_z \theta \rangle_{A,t} - \kappa \partial_z \langle \theta \rangle_{A,t}$$

$Nu = J/J_{conductive}$

Driven by:

$$Ra = \frac{\beta g \Delta L^3}{\kappa \nu}$$

Exact relation:

$$\tilde{\epsilon}_u = (Nu - 1) Ra Pr^{-2}$$

$$Pr = \nu / \kappa$$

$$\text{Scaling:} \quad Nu \propto Ra^{\beta}$$

TC

Conserved: angular velocity flux

$$J_\omega = r^3 \left[\langle u_r \omega \rangle_{A,t} - \nu \partial_r \langle \omega \rangle_{A,t} \right]$$

$Nu_\omega = J_\omega / J_{\omega,lam}$

Driven by:

$$Ta = \frac{d^2 r_a^6}{r_g^4} \frac{(\omega_1 - \omega_2)^2}{\nu^2}$$

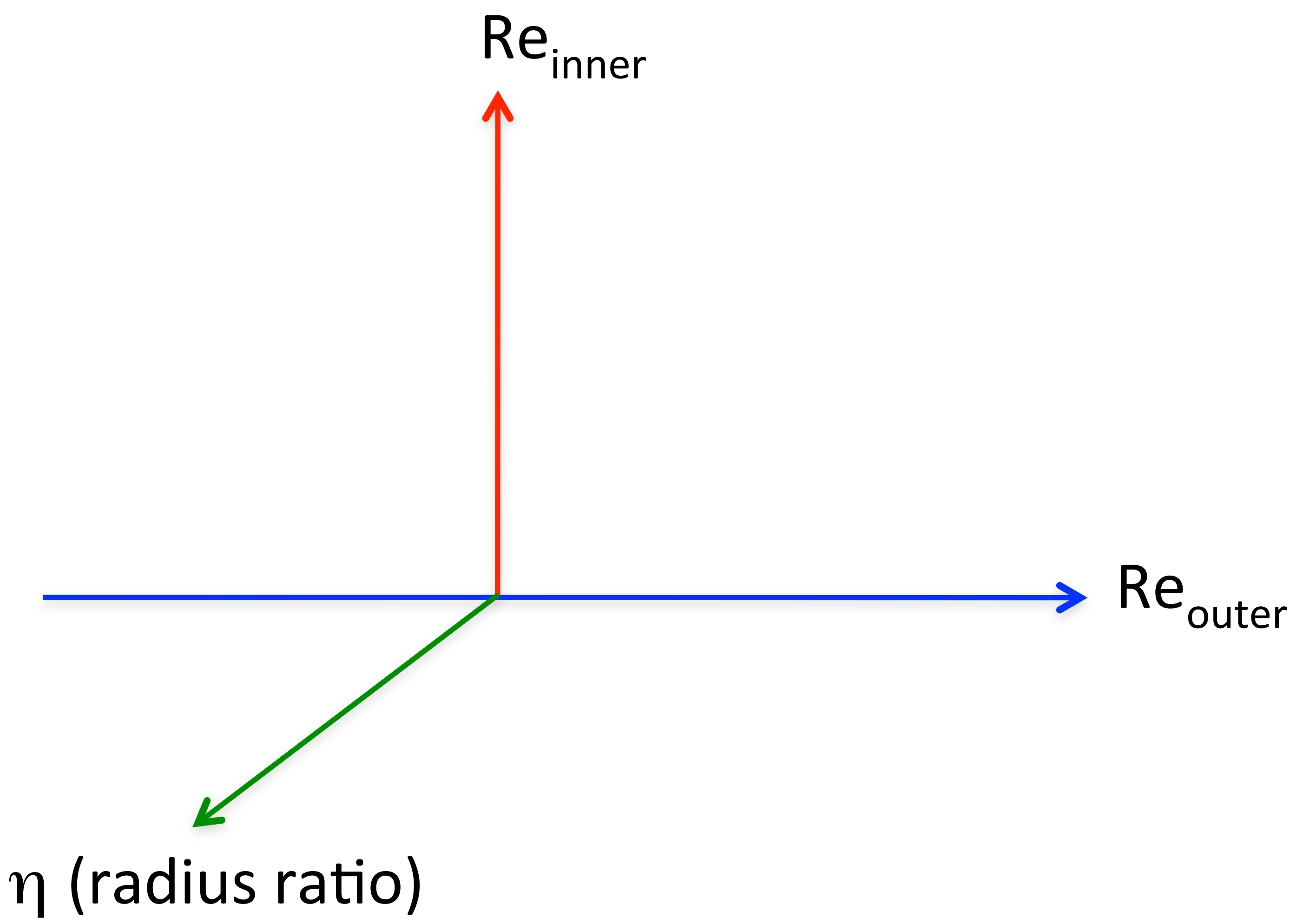
Exact relation:

$$\begin{aligned} \tilde{\epsilon}'_u &= \tilde{\epsilon}_u - \tilde{\epsilon}_{u,lam} \\ &= (Nu_\omega - 1) Ta \, \sigma^{-2} \end{aligned}$$

$$\sigma = (1 + \eta)^4 / (2\eta)^2$$

$$\text{Scaling:} \quad Nu_\omega \propto Ta^{\beta}$$

Parameter space



Shear Reynolds number $\mathrm{Re_s}$

$$Re_s = U_s \delta / \nu$$

$$Re_s = a_{bp}\sqrt{Re_i - Re_w}$$

$$\delta = \frac{a_{pb}}{\sqrt{Re_i - Re_w}} d$$

$$U_s = U_i - U_w$$

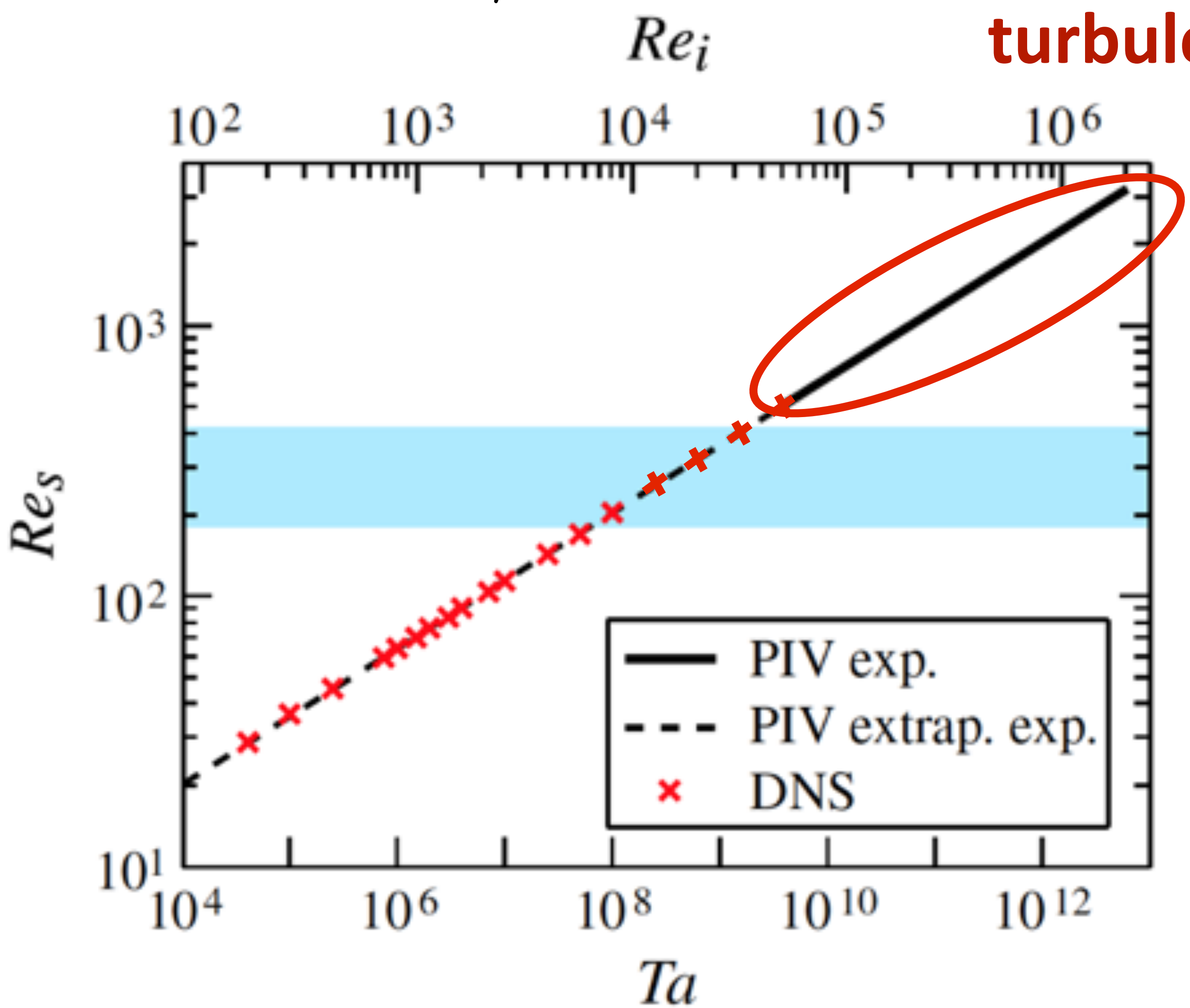
$$\frac{U_w}{U_i} \approx 0.05$$

Wind only small correction!

Shear Reynolds number Re_s

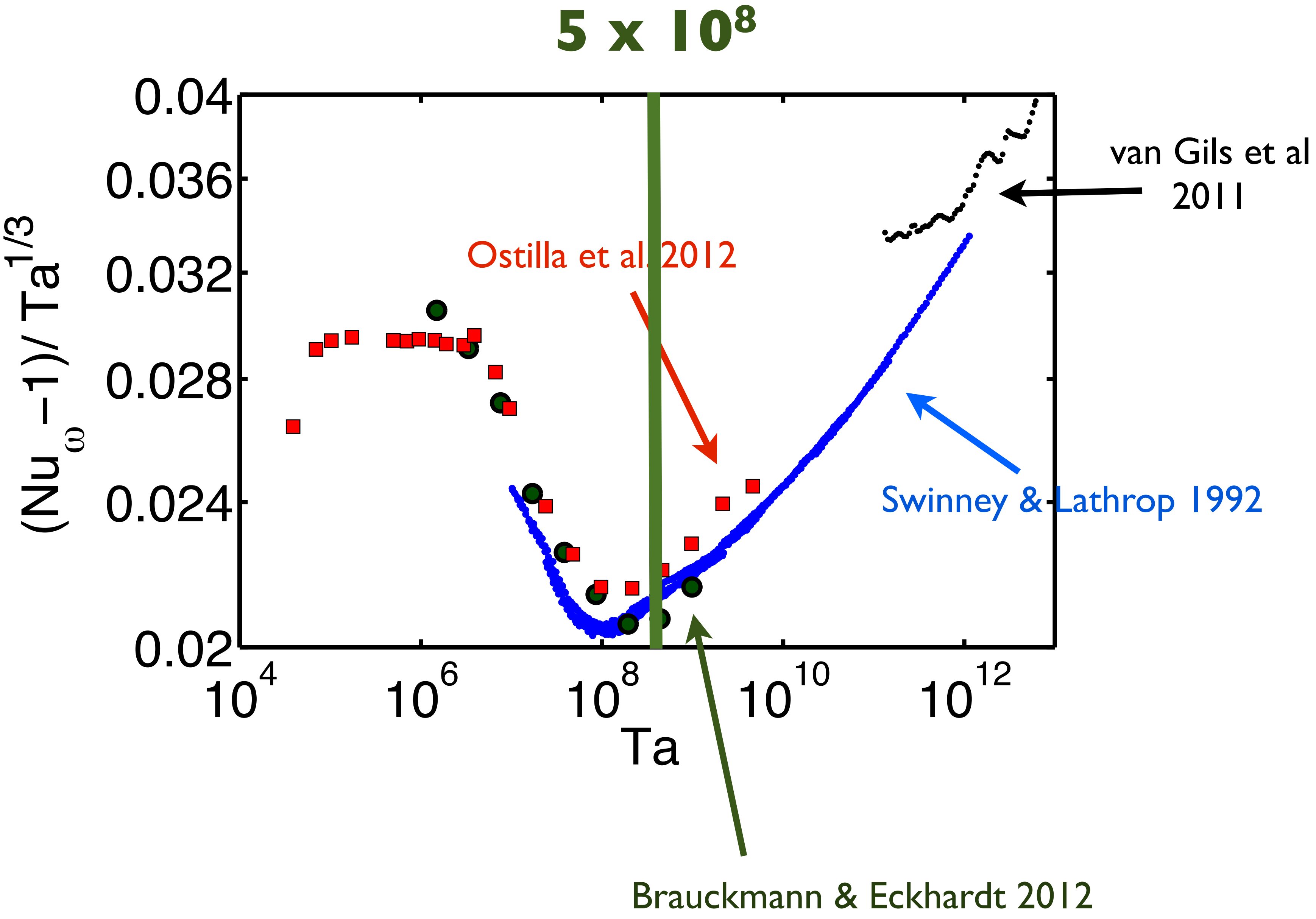
$$Re_s = U_s \delta / \nu$$

Flow in T^3C is in ultimate turbulent TC state!!!

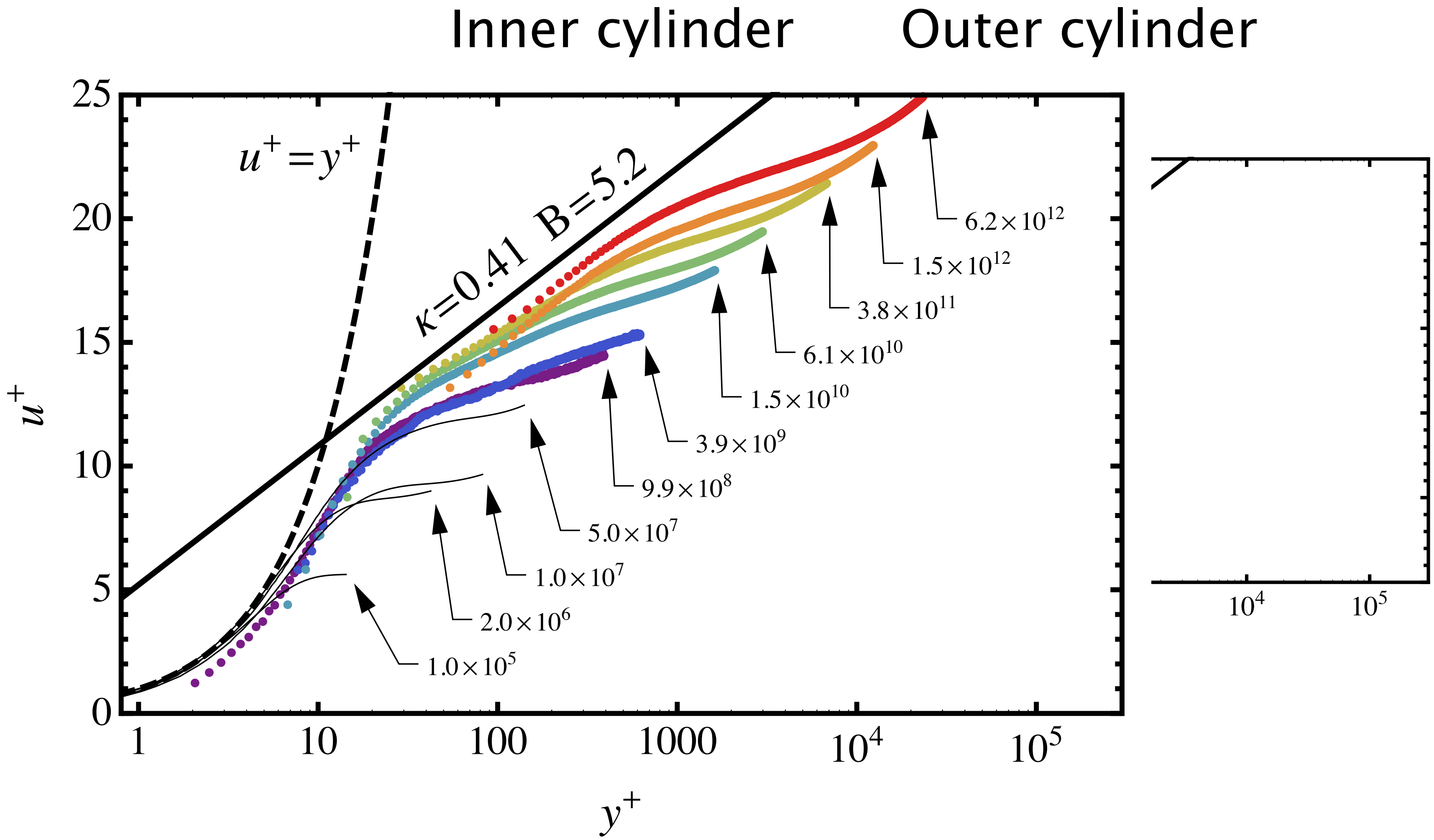


$$Ta_{crit} = 5 \cdot 10^8$$

Transition to ultimate regime



Boundary layer profiles in TC (by PIV)



Ostilla, Stevens, Grossmann, Verzicco, Lohse arXiv:1207:2290 (2012)

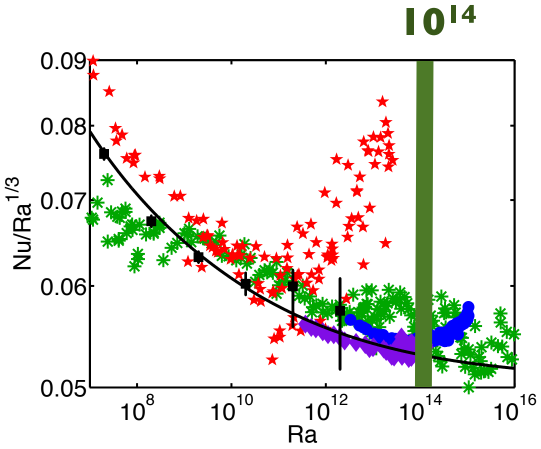
Perfect analogy RB vs TC even in ultimate regime,

but mechanical driving in TC much more efficient than thermal driving in RB

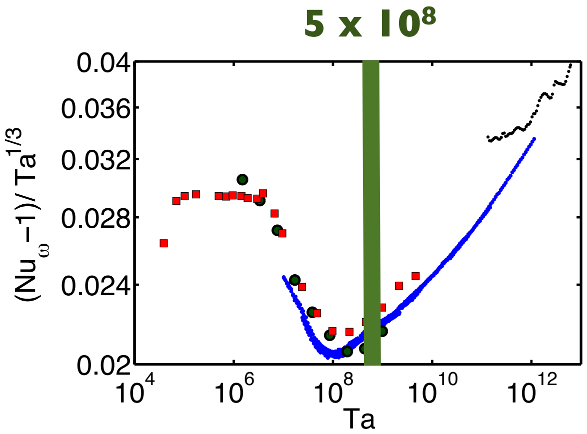
	RB	TC

Nu

RB

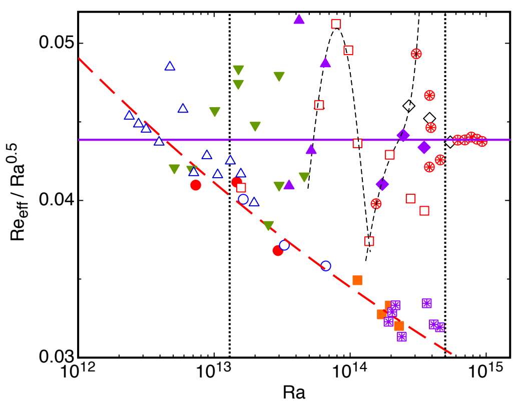


TC

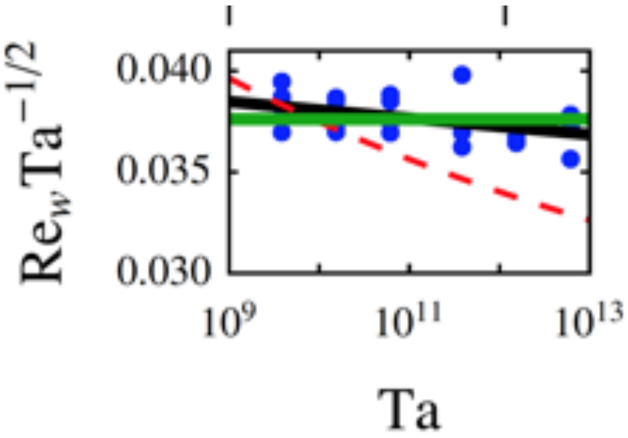


Re

RB

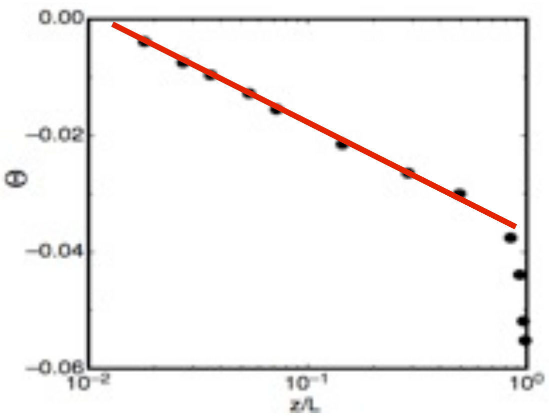


TC



profile in ultimate regime

RB



TC